THE YOUNG *ANALYST's* EXERCISE;

Or an Eaſy

INTRODUCTION TO *ALGEBRA.*

BEING

A Choice Collection of one Hundred
ALGEBRAICAL PROBLEMS, exhibiting
all that is curious in *Simple* and *Quadratic*
Adfected Equations.

The Whole illuſtrated
With various Methods of COMPUTATION,
as well by *plain Numbers* and *Geometry*, as by
Algebra; and calculated to the Capacity of young
Beginners.

The SECOND EDITION.
With large ADDITIONS.

By SAMUEL ASHBY,
Teacher of the MATHEMATICS. K

LONDON:
Printed for J. WILCOX, at *Virgil's* Head, o
poſite the *New Church* in the *Strand.*

M.DCCXLI.

BRITISH MUSEUM

TO

MARTIN CLARE, A.M.

AND

FELLOW

OF THE

ROYAL SOCIETY,

MASTER of the

ACADEMY

In *SOHO*-SQUARE.

This ſmall Treatiſe, deſign'd for the Uſe of *Schools*, being now made an eaſy *Introduction* to *Algebra*, and proper to exerciſe young Beginners, is moſt humbly *Dedicated*. By

Your Moſt Obedient,

and Humble Servant,

SAM. ASHBY.

Interest & property
in 28 houses in
Drapers Court Fleet
and Crescent an
Annuity of 300 per a
all pay J & Thos
in Lon[illegible] Dent
£3000 George Child
Leaves William Lye
Remains of the Ass

BRITISH MUSEUM

THE

PREFACE.

M*EETING with the following Propoſitions, publiſh'd by a certain* Latin *Author, well known to the* World *for his eminent Skill in* ALGEBRA, *but without their* Solutions; *and finding them a curious Collection, was prevailed upon by my own Inclination to draw out all their Anſwers. Which having finiſh'd, and ſhewn to ſome of my Acquaintance*

quaintance, very well skill'd in Mathematical *Learning, 'twas thought a Pity ſuch a Variety of uſeful Propoſitions ſhould lye hid in Oblivion, for that they would be of great Service, and very much inſtruct, and exerciſe the young* ANALYST, *and prepare him for higher Speculations in* Algebraical *Inquiries, if they were made public.*

I therefore complied with their Propoſal, and to render the Whole *more compleat, have now added the* Elements *of* Algebra *to it, to prevent the Learner running into unneceſſary Expences of too many Books.*

AND as the chief Deſign of this Treatiſe is to exerciſe the young ANALYST *in* ſimple *and* quadratic *Equations, ſuppoſing him but barely acquainted with the firſt Principles of* Algebra, *I have traced every Operation Step by Step, that thereby nothing might ſeem obſcure, and have likewiſe explain'd* *plain'd whatever I thought was not ſufficiently evident, leaſt the Learner ſhould ſpend his Time too much in uſeleſs Cogitations.*

AND notwithſtanding it is adapted to the Capacity of young Beginners, I doubt not but the more experienc'd Algebraiſt *will find ſomething in it worth his Obſervation too, For to many of the* Lineal Problems *I have not only given their* Analytical Inveſtigations, *but alſo their* Geometrical Conſtructions *and* Demonſtrations, *and the* Numerical Solutions *to many others.*

TO Problem XXXII *I have annexed three Examples, wherein I have ſhewn the* Geometrical *Reaſon of ambiguous Equations; with the original Derivation of the Method of compleating the Square; together with a ſhort Specimen of* Quadratic Equations, *in which is proved Algebraically, that the third Caſe of* Quadratic Adfected Equations, *is not univerſally ambiguous, as* Algebraiſts *generally take it to be, but only ſo in particular; with many other Particulars too numerous here to mention, which are beſt ſpecified by the Examples themſelves, to which I muſt refer the curious Reader.*

A HUNDRED *Algebraical Problems.*

THE principal Uſe of the Analytic Art, is to bring Mathematical Problems to Equations, and to exhibit thoſe Equations in the moſt ſimple Terms that can be. And the Rules required to bring out Equations to ſuch Problems, are chiefly theſe following, *viz.* *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*; *Involution* and *Evolution*. But if the Learner underſtand the firſt four Rules abovementioned, in whole Quantities and Fractions, ſo as to manage Equations by them; he will be ſufficiently qualified to anſwer many knotty and curious Queſtions without the Help of Involution or Evolution; as he will find by the I, II, III, IV, V, and many other of the following Operations.

Now as all Opperations in Arithmetic are perform'd by theſe *Arabian* Characters or Digits, *viz.* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; ſo in Algebra we perform all Operations by the Letters in the Alphabet, which repreſents the Things given as well as thoſe ſought: And becauſe algebraic Operations are founded on the ſame Principles with thoſe in common Arithmetic, and that the true *Sum*, *Difference*, *Product*, *Quotient*, *&c.* of algebraic Quantities are not to be expreſs'd as in common Numbers, we are oblig'd to make uſe of the following.

CHARACTERS.

$+$ *Addition* or *Plus.*
$-$ *Subtraction* or *Minus.*
$\times$ *Multiplication.*
$\div$ *Diviſion.*
$=$. *Equal.*

So that the Sum of any two or more Quantities, as a and b, is written thus: $a+b$, their Difference thus $a-b$, their Product $a\times b$, but generally written thus ab, and their Quotient thus $a\div b$, or thus $\frac{a}{b}$. When any two or more Quantities are on each Side this Sign, *viz.* $=$. they are equal to one another, ſo $a=b$, that is, a is equal to b. $a+x=c$, $a-x=d+y$, *&c.*

THE Uſe of theſe Signs being rightly underſtood, the Learner will not find Algebra ſo difficult to be attain'd, as perhaps he may imagine.

N. B. When there is any compound algebraic Quantity, as $a + b - c - d$, *it matters not which of the Terms ſtand firſt in the Quantity, providing it hath but its proper Sign prefixed to it; for the ſaid Quantity will ſtill retain the ſame Value if it ſtand thus,* $b - c + a - d$, *or in any other Poſition.*

OR *if ſeveral Quantities are multiplied together, as* xzy, *it is indifferent which of the Quantities ſtand firſt in the Product, for the ſaid Quantities may ſtand thus* zxy, *or thus* yxz, *&c.*

ALSO, *if the firſt Term of any compound Quantity be affirmative, it is always written without its Sign prefixed to it: As inſtead of writing the following Quantity with the affirmative Sign to the firſt Term, thus* $+x - y + z$, *we always write it thus,* $x - y + z$; *but a negative Quantity muſt always have its Sign prefixed to it, let its Situation be where it will; ſo in the foregoing Quantity if the Negative Term* $-y$ *be the firſt Term, it muſt be written thus* $-y + x + z$.

When an algebraic Quantity is multipled by a Number, as 12 x, that Number (12) is call'd the Co-efficient of that Quantity.

And when any Number is placed at the Top, or over any algebraic Quantity, thus: x^2, x^3, x^4, &c. that Number is call'd the *Index*, *Exponent* or *Uncia* of that Quantity, and ſhews what *Power* that Quantity is rais'd to.

Exam. If 2 be the Index of any Quantity, then that Expreſſion is call'd the Square of that Quantity, or ſecond Power, ſo x^2 is the *Square* or ſecond *Power* of x; and x^3 is the *Cube* or third *Power* of x; and alſo x^4, is the *Biquadrate* or fourth *Power* of x, &c.

When any Quantity, whether ſimple or compound, hath this Sign before it ($\sqrt{}$) it is call'd a *Surd* Quantity, and ſometimes *Radical*, and the Sign is call'd a Radical Sign, and ſhews that the ſquare Root of the Quantity to which it is prefix'd, is to be *extracted*; but if the Radical Sign hath any Number above it, thus ($\sqrt[3]{}$) it ſhews that there muſt be ſuch a Root extracted out of the Number to which it is prefix'd, as the Number above the Radical Sign contains *Units*. So, if there be 3, then there muſt be extracted the *Cube*, or third Root; if 4, the *Biquadrate*, or fourth Root, *&c.* that is, $\sqrt[3]{x}$ ſhews the Cube Root of x is to be extracted, and $\sqrt[4]{x}$ the Biquadratic Root, &c.

ADDITION.

CASE I.

IF the Quantities to be added together are of the ſame Species, and have like Signs; add their Co-efficients together, and to that Sum annex the Quantities, to which prefix their common Sign. Thus,

		(1)	(2)	(3)	(4)	(5)
	1	$4x + 7y$	$4x - 7y$	xy	$5x^2$	-64
	2	$3x + 2y$	$3x - 2y$	$9xy$	$10x^2$	-8
$1+2$	3	$7x + 9y$	$7x \quad 9y$	$10xy$	$15x^2$	-72

CASE II.

If the Quantities to be added together, are of the ſame Species, but have unlike Signs; ſubtract the Co-efficients from each other, and to their Difference join the Quantities with the Sign of the greater. Thus,

		(6)	(7)	(8)	(9)	(10)
	1	$4x + 7y$	$-4x - 7y$	$-xy$	$+5x^2$	64
	2	$-3x - 2y$	$3x + 2y$	$3xy$	$-10x$	-8
$1+2$	3	$x + 5y$	$-x - 5y$	$2xy$	$-5x^2$	56

N. B. $1+2$ *in the Margin ſhews that the firſt Step is added to the ſecond.*

OBSERVATION.

From the laſt Caſe it follows that if the affirmative Quantities and negative are the ſame, they will deſtroy one another; and their Sum will be equal to nothing. As is evident from the following Examples.

		(11)	(12)	(13)	(14)
	1	$4x + 7y$	$4xy$	$12x^2$	64
	2	$-4x - 7y$	$-4xy$	$-12x^2$	-64
1+2	3	0 0	0	0	0

Caſe III.

If the Quantities to be added together are of different Species, their Sum will be a compound Quantity; and can be no otherwiſe added but by their Signs. Thus,

		(15)	(16)	(17)
	1	x	x	$12a - 3x$
	2	y	$-y$	$-5y + 8z$
1+2	3	$x + y$	$x - y$	$12a - 3x - 5y + 8z$

CASE IV.

CASE IV.

IF the Quantities to be added together are Surd Quantities, and are of the ſame Kind; add the rational Part together as above ſhewn, and to the Sum annex the Surd Quantity. Thus,

		[18]	[19]	[20]	[21]
	1	$13\sqrt{xy}.$	$a\sqrt{ax-y}.$	$a-\sqrt[5]{x^2-y}.$	$\sqrt[3]{ax+y}.$
	2	$9\sqrt{xy}.$	$5a\sqrt{ax-y}.$	$b+\sqrt[5]{x+y}.$	$\sqrt[3]{ax+y}.$
1+2	3	$22\sqrt{xy}.$	$6a\sqrt{ax-y}.$	$a \quad b$	$2\sqrt[3]{ax+y}.$

CASE V.

IF the Surd Quantities are unlike, they are only to be added together according to their Signs. Thus,

		[22]	[23]	[24]
	1	$\sqrt{xy}.$	$3a\sqrt{bc}.$	$\sqrt[4]{x^2-xy+z}.$
	2	$\sqrt{az}.$	$\sqrt{x^2}.$	$\sqrt[7]{y^2+x^2}.$
1+2	3	$\sqrt{xy}.+\sqrt{az}.$	$3a\sqrt{bc}.+\sqrt{x^2}.$	$\sqrt[4]{x^2-xy+z}.\ \sqrt[7]{y^2+x^2}.$

SUBTRACTION.

TO ſubtract an Affirmative Quantity from an Affirmative, is the ſame as to add a Negative to an Affirmative.

And on the contrary, to ſubtract a Negative Quantity from an Affirmative; will be the ſame as to add an Affirmative Quantity to an Affirmative. Whence we have the following general

RULE.

Change all the Signs of the Quantities that are to be ſubtracted, or ſubtrahend, or ſuppoſe them to be chang'd; and then add them together according to the Rules of Addition, and that Sum will be the Remainder or Difference ſought. See the following Examples.

		(1)	(2)	(3)	(4)
	1	$4x+7y$	$4x+7y$	64	-64
	2	$3x+2y$	$-3x-2y$	-8	$+8$
1—2	3	$x+5y$	$7x+9y$	$+72$	-72

		(5)	(6)	(7)	(8)
	1	$3x+2y$	$-3x-2y$	8	8
	2	$4x+7y$	$4x+7y$	64	-64
1—2	3	$-x-5y$	$-7x-9y$	-56	72

		(9)	(10)	(11)
	1	$13\sqrt{xy}.$	$5a\sqrt{ax-y}.$	$\sqrt[4]{x^2-xy+z}.$
	2	$9\sqrt{xy}.$	$a\sqrt{ax-y}.$	$\sqrt[7]{y^2+x^2}.$
1--2	3	$4\sqrt{xy}.$	$4a\sqrt{ax-y}.$	$\sqrt[4]{x^2-xy+z}-\sqrt[7]{y^2+x^2}.$

N. B. 1—2 *in the Margin, ſhews that the ſecond Step is ſubtracted from the firſt.*

MUL-

MULTIPLICATION.

CASE I.

WHEN the Quantities have like Signs, multiply every Member of the Multiplicand by every Member of the Multiplier; and to every Term in the Product prefix the Sign +. Thus,

		(1)	(2)	(3)	(4)	(5)
	1	x	$-x$	$4x+7y$	$-4x-7y$	$-12-21$
	2	y	$-y$	$3x$	$-3x$	-9
1×2	3	xy	$+xy$	$12x^2+21xy$	$+12x^2+21xy$	$+108+189$

N. B. 1 × 2 *in the Margin ſhews that the firſt Step is multiplied by the ſecond.*

CASE II.

WHEN the Quantities to be multiplied have unlike Signs; multiply them as before, and to their Product prefix the Sign —. Thus,

		(6)	(7)	(8)	(9)
	1	$-x$	$4x+6y$	$-4x-6y$	$-12-21$
	2	y	$-3x$	$3x$	9
1 × 2	3	xy	$-12x-18y$	$-12x-18xy$	$-108-189$

THAT the Product of Quantities having like Signs is Affirmative, by the firſt Caſe, and unlike Negative by the ſecond Caſe, may be thus demonſtrated.

DEMONSTRATION.

As in Algebra there are but two principal Kinds of Quantities, *viz. Affirmative* and *Negative*; ſo the Rectangles of thoſe Quantities which have like Signs, muſt be contrary, or oppoſite, to thoſe having unlike; *Ergo*, *Plus* into *Plus* muſt produce a Rectangle contrary to *Plus* into *Minus*: And alſo *Plus* into *Minus* muſt be contrary to *Minus* into *Minus*; whence *Minus* into *Minus* muſt be the ſame as *Plus* into *Plus*. Now, no one can deny that *Plus* into *Plus* is Affirmative; conſequently *Minus* into *Minus* is Affirmative alſo. *Q. E. D.*

Or thus,

THAT + into +, and — into —, both produce an Affirmative Product, and + into —, or — into +, a Negative. Mr. WILLIAM GRIMMETT of *Deptford* in *Kent*, in an excellent Manuſcript of his own Writing, hath the following curious

DEMONSTRATION.

SINCE it is obſervable (ſaith that ingenious Gentleman) that no Quantity is either *Affirmative* or *Negative* naturally, but is only ſo with reſpect to ſome other Quantity of the ſame Kind, with which it is compared, and whoſe Sign is ſuppoſed to be fixed; we ſhall aſſume it as a firſt Principle, that a Product ariſing from two affirmative Quantities is an affirmative Product. Firſt then, we ſhall *demonſtrate* that an affirmative Multiplicand, multiplied by a negative Multiplicator, muſt produce a negative Product.

IN order to which, let us ſuppoſe, firſt, the Multiplicand to continue always the ſame, and then imagine the Multiplicator to be conſtantly leſſening, and it is evident the Products muſt be leſſening in the ſame Proportion: For when the Multiplicator becomes nothing, the Product will become nothing alſo; and therefore, when the Multiplicator becomes leſs than nothing, or negative, the Product muſt conſequently become leſs than nothing, or negative with it, although the Multiplicand do always continue an affirmative Quantity. Therefore an affirmative Quantity multiplied by a negative Quantity will produce a negative Product. As by the following Diagram is evident.

Multiplicand.	+3	+3	+3	+3	+3	+3	+3 .
Multiplicator.	+3	+2	+1	+0	−1	−2	−3 .
Product.	+9	+6	+3	+0	−3	−6	−9 .

Q. E. D.

Secondly. AND ſince it is all one as to the Products, which of the two Quantities is made the Multiplicand, it follows alſo, that a negative Quantity multiplied by an affirmative one, muſt produce a negative Product.

Q. E. D.

Thirdly. WE are now in the laſt Place to demonſtrate, that the Product of a negative Quantity multiplied by a negative Quantity is affirmative.

Which is thus,

LET us ſuppoſe the Multiplicand to be a negative Quantity, and to continue always the ſame, but the Multiplicator an affirmative Quantity, and conſtantly leſſening, as in the firſt Caſe. Then it is evident that whilſt the Multiplicator is an affirmative Quantity, the Product will be negative (*by the foregoing*). And whilſt the Multiplicator leſſens or becomes leſs affirmative, the Product will be proportionably increaſing, and ſo become leſs

Negative; and when the Multiplicator becomes nothing, the Product will be nothing alſo, or will be in a limit between Affirmative and Negative, and conſequently whenever the Multiplicator becomes Negative, the Product will become Affirmative. Therefore — into — produces +. See the following Diagram.

Multiplicand.	−3	−3	−3	−3	−3	−3	−3	.
Multiplicator.	+3	+2	+1	0	−1	−2	−3	.
Product.	−9	−6	−3	−0	+3	+6	+9	

Q. E. D.

CASE III.

If compound Quantities are to be multiplied by compound Quantities, multiply the Multiplicand by every Term in the Multiplier, and add all their Products together, by the Rules of Addition, and that Sum will be the Product required.

(10)

Multiply	$x + y$
by	$z + u$
	$xz + zy$
	$\qquad xu + uy$
Product.	$xz + zy + xu + uy$

(11)

$$\begin{array}{rl} \textit{Multiply} & 5xy + 13zu \\ \textit{by} & 8 - 15a \\ \hline & 40xy + 104zu \\ & \quad - 72axy - 195azu \\ \hline \textit{Product.} & 40xy + 104zu - 72axy - 195azu \\ \hline \end{array}$$

(12)

$$\begin{array}{rl} \textit{Multiply} & x + y \\ \textit{by} & x - y \\ \hline & x^2 + xy \\ & \quad - xy - y^2 \\ \hline \textit{Product.} & x^2 - y^2 \\ \hline \end{array}$$

THIS laſt Example ſhews that the Rectangle of the Sum and Difference of any two Quantities, is equal to the Difference of their Squares, *which is worth Notice.*

CASE IV.

CASE IV.

If the Quantities to be multiplied are Surd Quantities, and of the ſame Kind, multiply the Quantities under the Radical Sign as is taught above, and to their Product prefix the Radical Sign. Thus

	(13)	(14)
Multiply	$\sqrt{ax-x^2}$.	$\sqrt[3]{x^2+xy+y^2}$.
by	$\sqrt{zy}$.	$\sqrt[3]{x-y}$.
Product	$\sqrt{axyz-x^2yz}$.	$\sqrt[3]{x^3-y^3}$.

CASE V.

If Surd Quantities of the ſame Kind, that is, if they are both Square Roots, or both Cube Roots, or both Biquadratic Roots, &c. are multiplied into Numbers or Species; multiply the Surd Quantities as before, and to their Product prefix the Product of the ſaid Numbers or Species. Thus

	(15)	(16)
Multiply	$ab\sqrt{x+y}$.	$a-2b\sqrt[4]{x-y^3}$.
by	$a^2e\sqrt{x-y}$.	$cd\sqrt[4]{x^2}$.
Product.	$a^3be\sqrt{x^2-y^2}$.	$acd-2bcd\sqrt[4]{x^3-x\,y^3}$.

DIVISION.

Diviſion of Species is the Reverſe to Multiplication.

IN Diviſion there are always two Quantities given to find a third; of which, that Quantity which is to be divided, is call'd the *Dividend*; that by which it is to be divided, is call'd the *Diviſor*; and the third Quantity which comes out by Diviſion, is call'd the *Quotient*.

2 WHENCE the Diviſor and Quotient multiplied into one another is always equal to the Dividend; therefore, as in Multiplication like Signs gives +, and unlike —, ſo in Diviſion when the Diviſor and Dividend have like Signs, the Quotient will be +, but if unlike, then the Quotient will be —.

CASE I.

IF the Quantities are ſimple, and have no Co-efficients, their Quotient will appear by Inſpection only. Thus,

(1) (2) (3) (4)

$\frac{xy}{x} = y. \quad \frac{-xy}{-x} = y. \quad \frac{-xy}{x} = -y. \quad \frac{xy}{-x} = -y$

In the firſt and ſecond Examples the Quotient is y, and in the third and fourth the Quotient is $-y$.

(5) (6) (7) (8)

$$\frac{x^2y}{x}=xy;\ \frac{-x^2y}{xy}=-x.\ \frac{x^2y}{-y}=-x^2.\ \frac{-x^2y}{-x^2}=y.$$

Case II.

When the Diviſor and Dividend are different Quantities, ſet the Diviſor underneath the Dividend with a Line betwixt them, as is ſhewn in the foregoing Examples, and that Form will be the Quotient ſought. Thus,

(9) (10) (11)

$$\frac{xy}{z},\ \frac{xy-ax}{bc+d},\ \frac{x^3-3ax^2+3a^2x-a^3}{z^2+2zy+y^2}.$$

CASE III.

WHEN a compound Quantity is to be divided by a compound Quantity, ſet down the Diviſor and Dividend as in common Arithmetic, and proceed according to the Methods there taught, and you will have the Quotient required.

(12)

Diviſor. *Dividend.* *Quotient*

$$a-y\,)\,a^2x-3aby-axy+3by^2\,(ax-3by.$$

$$a^2x-axy$$

$$-3aby-3by^2$$

$$-3aby-3by^2$$

$$0 \quad 0$$

The Performance of this Operation is thus. Firſt, I ſeek how often *ax* the firſt Term in the Diviſor is contain'd in a^2x, the firſt Term in the Dividend, and I find it *ax*, this I place in the Quotient, and it is an Affirmative Quantity, becauſe *a* and a^2x are Affirmative; then I multiply $a-y$, the Diviſor, by *ax* the Quotient Figure, as in common Arithmetic, and the Product is $a^2x-a\;y$ which I write under the Dividend, and being ſubtracted from thence, there remains $-3aby-3by^2$; then I ſeek how often a, the firſt Term in the Diviſor, is contain'd in $-3aby$, the firſt Term in the Remainder, and I find it $-3by$, which I place in the Quotient, and having multiplied and ſubtracted as before, there remains (0), whence I conclude the Quotient to be $ax-3by$.

To prove this Operation multiply the Quotient by the Diviſor, and if it is right, the Product will be the ſame with the Dividend. Thus,

$$\begin{array}{rl}
\textit{Multiply} & ax-3by \\
\textit{by} & \underline{a-y} \\
 & a^2x+3aby \\
 & \quad\; \underline{-xy+3by^2} \\
\textit{Product.} & \underline{a^2x-3aby-axy+3by^2}.\ \textit{Proof.}
\end{array}$$

(13)

$$\begin{array}{l}
x-y\,)\ x^3-y^3\ (x^2+xy+y^2. \\
\qquad\quad \underline{x^3-x^2y} \\
\qquad\qquad x^2y-y^3 \\
\qquad\qquad \underline{x^2y-xy^2} \\
\qquad\qquad\qquad xy^2-y^3 \\
\qquad\qquad\qquad \underline{xy^2-y^3} \\
\qquad\qquad\qquad\ 0 \quad\ 0
\end{array}$$

CASE IV.

CASE IV.

If the Diviſor and Dividend are both Surd Quantities of the ſame Denomination, that is, both Square Roots, Cube Roots, &c. divide the Quantity under the Radical Sign, as above, and to their Quotient prefix the Radical Sign, and you will have the Quotient required. Thus,

(14)

$$\frac{\sqrt{xy.}}{\sqrt{x.}}=\sqrt{y.}\ \text{the Quotient.}$$

(15)

$\sqrt{x^2-y^2.})\ \sqrt{x^4-y^4.}\ (\sqrt{x^2+y^2.}$ This Opperation without the Radical Signs runs thus:

$$\begin{array}{l}
x^2-y^2)\ x^4-y^4\ (x^2+y^2 \\
\qquad\quad \underline{x^4-x^2y^2} \\
\qquad\qquad\quad x^2y^2-y^4 \\
\qquad\qquad\quad \underline{x\,y^2-y^4} \\
\qquad\qquad\quad\ 0 \qquad 0
\end{array}$$

the Quotient, which put under the Radical Sign will ſtand thus:

$\sqrt{x^2+y}$. the ſame as before.

CASE V.

CASE V.

If the Surd Quantities are multiplied into rational Quantities, divide the Surd Quantities as before, and to their Quotient prefix the Quotient of the Rational Quantities. Thus,

(16) (17)

$$\frac{12\sqrt{xy}\,.}{3\sqrt{x}\,.}=4\sqrt{y}\,.\qquad \frac{a\sqrt{x^4-y^4}\,.}{6\sqrt{x^2-y^2}\,.}=\frac{a}{6}\sqrt{x^2+y^2}\,.$$

The Learner may now, if he pleaſes, proceed to the Solutions of the firſt fourteen Queſtions, for in them he will have no Occaſion for the two following Rules.

INVOLUTION.

Involution *is the multiplying of any propos'd Root whether ſimple or compound, into itſelf, any Number of Times.*

THE Number propos'd to be involv'd is call'd the *Root* or firſt *Power*, the firſt Multiplication is call'd the Square or ſecond Power, the ſecond Multiplication is call'd the *Cube* or third *Power*, &c. as in theſe following.

	1	x	$-x$	the Root or firſt Power.
1@2	2	xx	$+xx$	Square or ſecond Power.
1@3	3	xxx	$-xxx$	Cube or third Power.
1@4	4	$xxxx$	$+xxxx$	Biquadrat or fourth Power.

THIS (@) is the Note of *Involution*, and at the ſecond Step, 1@2, ſhews that the firſt Step is rais'd to the ſecond Power or ſquar'd; 1@3, at the third Step ſhews the firſt Step rais'd to the *third* Power or cub'd, *&c.*

Exam. 2. Let it be required to raiſe $x+y$ to the *ſecond* Power or Square?

$$\begin{array}{ll} x+y & \\ x+y & \\ \hline \left.\begin{array}{l} x^2+xy \\ \quad\;\; +xy+y^2 \end{array}\right\} & \text{Add.} \\ \hline x^2+2xy+y^2 & \text{the Square of } x+y. \end{array}$$

Exam. 3. Let it be required to raiſe $x-y$ to the *ſecond* Power or Square?

$$\begin{array}{ll} x-y & \\ x-y & \\ \hline \left.\begin{array}{l} x^2-xy \\ \quad\;\; -xy+y^2 \end{array}\right\} & \text{Add.} \\ \hline x^2-2xy+y^2 & \text{the Square of } x-y, \text{ which} \end{array}$$

differs from the former only in the Signs + and —.

Exam. 4. Let it be required to raiſe $x+y$ to the *third* Power or Cube?

$$\begin{array}{l} x+y \\ x+y \\ \hline \left.\begin{array}{l} x^2+xy \\ \quad\;\; +xy+y^2 \end{array}\right\} \text{Add.} \\ \hline x^2+2xy+y_2 \;\text{ the Square of } x+y. \\ x+y \\ \hline \left.\begin{array}{l} x^3+2x^2y+xy^2 \\ \quad\;\; +x^2y+2xy^2+y^3 \end{array}\right\} \text{Add,} \\ \hline x^3+3x^2y+3xy^2+y^3 \;\text{ the Cube of } x+y. \\ \hline \end{array}$$

Exam. 5. Let it be required to raiſe $x-y$ to the *third* Power or Cube?

$$\begin{array}{l} x-y \\ x-y \\ \hline \left.\begin{array}{l} x^2-xy \\ \quad\;\; -xy+y^2 \end{array}\right\} \text{Add} \\ \hline x^2-2xy+y^2 \;\text{ the Square of } x-y. \\ x-y \\ \hline \left.\begin{array}{l} x^3-2x^2y+xy^2 \\ \quad\;\; -x^2y+2xy^2-y^3 \end{array}\right\} \text{Add} \\ \hline x^3-3x^2y+3xy^2-y^3 \;\text{ the Cube of } x-y. \\ \hline \end{array}$$

which is the ſame as the former, except that there every Term is affirmative, and in this every other Term is negative.

Exam. 6. What is the Square of $5a+x-y-7z$?

$$\begin{array}{l}
5a+x-y-7z \\
\underline{5a+x-y-7z} \\
25a^2+5ax-5ay-35az \\
\quad +5ax+x^2-xy-7xz \\
\qquad -5ay-xy+y^2+7yz \\
\qquad\quad -35az-7xz+7yz+49zz, \\
\hline
25a^2+10ax-10ay-70az+x^2-2xy-14xz+y^2+14yz+49z^2 \\
\hline
\end{array}$$

the Square ſought.

From the foregoing Example it appears, that the Square of any compound Quantity, contains ſo many perfect Squares as there are Terms in the Quantity, and ſo many double Rectangles as there are Terms beyond that Quantity. That is, the Square of $5a+x-y-7z$ is equal to the Square of $5a$ and the double Rectangle of $5a$ into $+x-y-7z$, the Square of x and the double Rectangle of x into $-y-7z$, the Square of $-y$ and the double Rectangle of $-y$ into $-7z$, and the Square of $-7z$. Whence it will be very eaſy to raiſe any compound Quantity to the ſecond Power without a previous Multiplication, for the Square of $x+y-z$ will be $x^2+2xy-2xz+y^2-2yz+z^2$.

fitly expreſsed

If any Radical Quantity is to be involv'd to a Power whoſe Index is oppoſite to that of the Radical Sign, it is but taking the Radical Sign away, and you will have the Power required.

Exam. 7. What is the Square or ſecond Power of $\sqrt{a^2b + ax^2 - dz^4.}$?

The Radical Sign being taken away, the Square required will be $a^2b + ax^2 - dz4$.

So the Cube of $\sqrt[3]{x^5 - a^2x^3 + bz.}$ will be $x^5 - a^2x^3 + bz$; and the Biquadrate or fourth Power of $\sqrt[4]{a^5b^2 + bx^2 - bd}$ is $a^5b^2 + bx^2 - bd$.

EVOLUTION.

Evolution is the Reverſe to *Involution*, for whereas *Involution* teaches to raiſe any given Quantity to any Power propos'd, *Evolution* teaches to extract any aſſign'd Root of any Quantity given.

If the Quantity given be a ſimple one, the Root propos'd to be extracted will be diſcover'd at firſt Sight. As appears by the following Examples.

(1)

	1	x^6
1 ɯ $\overline{2}$	2	x^3
1 ɯ $\overline{3}$	3	x^2
2 ɯ $\overline{3}$	4	x
3 ɯ $\overline{2}$	5	x

(2)

	1	$64x^6$
1 ɯ $\overline{2}$	2	$8x^3$
1 ɯ $\overline{6}$	3	$2x$
2 ɯ $\overline{3}$	4	$2x$

(3)

	1	$ax-y$
1 ɯ $\overline{2}$	2	$\sqrt{ax-y}$.
1 ɯ $\overline{3}$	3	$\sqrt[3]{ax-y}$.
1 ɯ $\overline{4}$	4	$\sqrt[4]{ax-y}$.

The laſt Example ſhews that when the given Quantity hath not ſuch a Root as is required to be extracted, then you muſt put the Radical Sign over it, and the Index of the Root to be extracted over that, and that will be the Root ſought.

N. B. *When any abſolute Number is regiſtered in the Margin, you draw a Line over it, to diſtinguiſh it from the Numbers of the Steps. As* $\overline{2}$ *in the ſecond Step of the firſt Example and the reſt.*

When the Quantity given to be extracted or evolv'd is a compound one, it very often proves very difficult to diſcover the Root thereof, even by thoſe who are tolerable good Algebraiſts, and thereſore I preſume it will be of little or no Service to the Learner to give him any Examples of this Kind at preſent, the foregoing is ſufficient for any Opperation he will meet with in this Book; except when there happens to be ſuch compound Quantities as $x + 2ax + a^2$, or $x^2 - 2ax + a^2$, whoſe ſquare Root is as eaſily diſcovered as if they were ſimple Quantities. For the Root of the former is $x + a$, and of the latter $x - a$, as appear from Involution.

Having thus laid down the firſt Principals of *Algebra*, I will now proceed to the following Hundred Queſtions and Anſwers; but firſt ſhall premiſe theſe three Particulars, *viz.* That in all Queſtions whatſoever theſe three Things are chiefly to be obſerved; *firſt*, the *Data* or Things given; *ſecondly*, The *Quæſita* or Things ſought; and *thirdly and laſtly*, The Condition or Form of a Queſtion; which three Diſtinctions are fully illuſtrated in the following Solution.

PROBLEM I.

To find a Number, which being multiplied by 3, ſubtracting 5 from the Product, and the Remainder divided by 2, if the Number ſought be added to the Quotient, that the Sum may be 40.

For a perfect Idea of this, or any Queſtion propoſed, divide it into theſe three Particulars; *firſt*, The *Data*, which in this Problem are theſe Numbers 3, 5, 2, and 40; *ſecond*, The *Quæſita*, which is the Number to be found; and *third*, The *Condition* or *Form* of the Queſtion, which in this Problem runs thus. Which being multiplied by 3, ſubtracting 5 from the Product, and the Remainder divided by 2, if the Number ſought be added to the Quotient, that the Sum may be 40.

Now to ſolve this Queſtion, for the Numbe ſought put any Symbole or Letter, as for inſtance x; which by the Condition of the Problem, muſt *firſt* be multiplied by 3, and $3x$ will be the Product; *ſecondly*, Subtract 5 from that Product, and the Remainder will be $3x-5$; *thirdly*, Divide the ſaid Remainder by 2, and the Quotient will be $\frac{3x-5}{2}$; *fourthly*, *and laſtly*, Add the Number ſought, *viz.* x to this Quotient $\frac{3x-5}{2}$, and the Sum is $\frac{3x-5}{2}+x$; which Sum by the Queſtion is equal to 40. And thus the Queſtion is tranſlated out of the Verbal into a Symbolical Expreſſion, and therefore is brought to this

Equation	1	$\frac{3x-5}{2}+x=40$. And to find the Value of x proceed thus,
1 $\times \overline{2}$	2	$3x-5+2x=80$
2 $+\overline{5}$	3	$5x=85$
3 $\div\overline{5}$	4	$x=17$ the Number which was to

be found, or the Value of x required.

W. W. D.

FROM the Operation aforegoing the Learner may obſerve, that to the Solution of Queſtions, which only regard Numbers, or the abſtracted Relations of Quantities, there is ſcarce any thing elſe required, than that the Problem be tranſlated out of the *Engliſh*, or any other Tongue it is propoſed in, into the Algebraical Language, that is, into Characters fit to denote our Conceptions of the Relations of Quantities. For when a Queſtion is expreſſed Analytically, the Calculation or Form of the Reduction, for the moſt part, is ſeldom any thing more, than the Condition of the Problem repeated backward; and thus the Condition or Form of a Queſtion, in a manner, may ſerve for a Rule to inveſtigate the unknown Quantity by.

But the moſt commodious and univerſal Solutions of Algebraical Problems are generally performed by Symboles or Letters repreſenting the Quantities given as well as thoſe required. Thus, put $3=a$, $5=b$, $2=c$, and $40=d$; and ſuppoſing x ſignifies the Number ſought as before; then inſtead of this Numeral Equation $\frac{3x-5}{2}+x=40$, we ſhall have this Literal one $\frac{ax-b}{c}+x=d$, from whence the Quantity x will be found

$$\text{Thus,}\quad \begin{array}{ll|l|l} & & 1 & \frac{ax-b}{c}+x=d. \\ 1 & \times c & 2 & ax-b+cx=cd \\ 2 & +b & 3 & ax+cx=cd+b \\ 3 & \div a+c & 4 & x=\frac{cd+b}{a+c} \end{array}$$

, the Value of x required.

By this Method of Computation it appears, that not only the unknown Quantity is inveſtigated, but a general Theorem or Rule is diſcovered for ſolving all Queſtions of the ſame Nature; for the Letters a, b, c, and d, may be put for any other Numbers than what they here repreſent. Therefore this Equation $\frac{cd+b}{a+c}=x$ is a general Theorem for finding the Number ſought to all Queſtions having the ſame *Data* and *Quæſita* as that above.

N. B. THAT the aſſumed Quantities in all Operations are eſteemed the very ſame as tho' really known: For any of the known Quantities may be brought to one ſide of the Equation, and made equal to all the reſt, by the ſame Principals of *Algebra* as the unknown Quantities are calculated. To explain which, let any of theſe known Quantities a, b, c, in this Equation $\frac{ax-b}{c}+x=d$, as b for inſtance, be cleared and brought to one ſide the Equation, and all the reſt to the other. The Equation being thus

Stated, | 1 | $\frac{ax-b}{c}+x=d$. Work as follows,

1 $\times c$ | 2 | $ax-b+cx=cd$
2 $+b$ | 3 | $cd+b=ax+cx$
3 $-cd$ | 4 | $b=ax+cx-cd$.

But if it had been required to have brought out c, then

2	$-cx$	5	$cd-cx=ax-b$
5	$\div d-x$	6	$c=\frac{ax-b}{d-x}$. Or a, Then,
5	$+b$	7	$ax=cd-cx+b$
7	$\div x$	8	$a=\frac{cd-cx+b}{x}$
Or 2	$+b-cx$	9	$ax=cd+b-cx$
9	$\div x$	10	$a=\frac{cd+b-cx}{x}$

Hence it is, that Algebraiſts ſuppoſe thoſe Things given, which notwithſtanding are the ſole Objects of their Inquiry, until by Reduction they become equal to ſome certain Poſition of the given Quantities; and then the true Value of ſuch feigned Numbers are diſcovered.

FARTHER; to ſtrengthen the young Analyſt's Idea in Algebraical Computation, he muſt underſtand, that the Method of aſſuming Letters for the unknown Quantities, is no more than gueſſing the Value of ſuch Quantities at randum. So in the foregoing Propoſition, inſtead of making uſe of x, I may ſuppoſe the Quantity ſought to be any Number at Pleaſure, and the Queſtion being ſtated with that Number and thoſe given, according to the Condition or Form of the Problem, the true Quantity required will then come out by the ſame Laws of *Algebra*, as tho' the Number ſought was repreſented by any Letter in the Alphabet, or other Symbole. Thus, for the Number to be found put 9, and by the Condition of the Queſtion, there will be had this numeral

$$\begin{array}{l|l|l}
\text{Equation} & 1 & \frac{\overline{3\times9}-5}{2}+9=40. \\
1 \qquad \times\overline{2} & 2 & \overline{3\times9}-5+\overline{2\times9}=80 \\
2 \qquad +\overline{5} & 3 & \overline{3\times9}+\overline{2\times9}=85 \\
3 \ \div\overline{3}+\overline{2} & 4 & 9=\frac{85}{3+2}=17.
\end{array}$$

Here the *Calculus* is the very ſame, and brings out the ſame Concluſion, as either of the foregoing Methods, and likewiſe ſhews the Suppoſition to be falſe; for inſtead of 9 I ſhould have gueſt 17. Whence we may conclude, that the ultimate Deſign of Algebraical Operations, is no more than to diſcover the Truth from falſe Suppoſitions.

MOREOVER ſeeing my Suppoſition was too little, I may ſupply that Defect with x, and ſo call the Number ſought $9+x$; and having expreſs'd the Problem Algebraically, there will be obtain'd this

Equation	1	$\frac{27+3x-5}{2}+9+x=40.$
1 $\times 2$	2	$27+3x-5+18+2x=80$
2 $+$	3	$5x=40$
3 $\div 5$	4	$x=\frac{40}{5}=8.$ Wherefore $9+8=17$ the

Number which was to be found, the ſame as before.

From what is here ſpecified the Artiſt may obſerve, that let the aſſumed Number be what it will, there will ſtill reſult the ſame Concluſion to the Queſtion propoſed; ſo that it matters not whether the whole or a part of the Quantity, or ſome other Number, be made the Scope of our Inquiry, ſince they all bring out the true Anſwer to the Problem.

But let the unknown Quantity be what it will, the Algebraiſt muſt accomodate it, ſo as to bring out the moſt ſimple Equation poſſible. And ſince from the various Species of Problems different Equations ariſe, ſo thoſe wherein only one Power of the unknown Quantity is contained, as in theſe following,

Viz.	$ax+cx=cd+b$	I.	Theſe Numbers ſhew the Operation each Equation is taken from.
	$25x^2+16x^2=16a$	XVI.	
	$x^2=2ab.$	XXXVI.	
	$x^3=ab^2$, &c.	XXXVII.	

are called Simple Equations. An thoſe wherein different Powers of the Quantity ſought are contained, as for Example,

$x + ax = ab.$	LXIX.
$bx^2 - cx^2 + 2acx = a^2c.$	LXXVI.
$x^4 - cx^2 = ab$, &c.	LXXVII.

are called compound or adfected Equations.

As there ariſe different Powers of the unknown Quantity in Algebraical Operations, ſo there are different Forms of both ſimple and compound Equations; for $ax - cx = cd + b$, is called a pure Simple Equation, $25x^2 + 16x^2 = 16a$ or $x^2 = 2ab$ a Simple Quadratic, and $x^3 = ab^2$ a Simple Cubic, &c. And in compound Equations, thoſe wherein the unknown Quantity is found of no more than two different Powers, whoſe Indices are double to each other, are called adfected, and is either Quadratic, or Biquadratic, &c. according to the higheſt Dimentions of the Quantity ſought. That is, $x^2 + ax = ab$ or $bx^2 - cx^2 + 2acx = a^2c$, is called a Quadratic Adfected Equation, but $x^4 - cx.^2 = ab$ is called a Biquadratic Adfected, &c.

Having thus defined the different Kinds of Equations contained in the following Pages, I will now ſhew the Learner how the ſaid Equations are brought to a Solution. And firſt then, for Simple Equations take this general

RULE.

R U L E.

Extract ſuch a Root of the Equation, whoſe Index ſhall be the Power of the unknown Quantity, and it is done.

As per Example, In this Equation $x^2 = 2ab$, becauſe the unknown Quantity x is ſquared, I extract the ſquare Root, and there comes out $x = \sqrt{2ab}$. So alſo in this Equation $x = ab^2$, by extracting the Cube Root we have $x = \sqrt[3]{ab}$, and ſo on.

I SHALL omit the Rule for ſolving Quadratic adfected Equations at preſent, until the Learner has been ſome Time exerciſed in Simple Equations; becauſe the Learning of Rules without having an immediate Uſe for them, very often ſerves more to perplex than inſtruct the Artiſt; for tho' a Learner be never ſo expert in the Elements of *Algebra*, yet when he comes to the Application of theſe Principals in the Solution of Propoſitions (except he is Maſter of a good Genius and ſtrong Memory) he will find ſuch a Crowd of confuſed Ideas in his Mind, that if he is not helped by the Inſtruction of a Maſter or ſome Acquaintance, he will be ready to cry out *neplus ultra*. But by proceeding according to the Methods laid down in the foregoing Pages, and obſerving well the following Operations, he will in a great Meaſure, be freed from ſuch Inconveniencies, and find, that *Algebra* is no more difficult to attain, than common Arithmetic. For both are founded on the ſame Principals; and aim at the ſame End. And differ from each other only in this Particular, that vulgar Arithmetic begins its Computation from the Quantities given, and ſo proceeds forwards to thoſe that are required, whereas *Algebraic* Computations proceed in a retrograde Order, *viz.* from the Quantities ſought, as if they were given, to the Quantities given, as if they were ſought, to the End that we may ſome way or other come to a Concluſion or Equation, from which we may bring out the Quantity ſought. Which the Reader may obſerve from the Solution to the tenth Queſtion, and ſeveral others in the preceding Pages.

PROBLEM II.

To find a Number, which being multiplied by 12, *and* 48 *added to the Product, as much may be produc'd, as if the ſame Number ſought were multiplied by* 18.

Put 12=*a*, 48=*b*, 18=*c*; and let the Number ſought be called *x*. Then by the Queſtion we ſhall

	have	1	$ax+b=cx$
1	$-ax$	2	$cx-ax=b$
2	$\div c-a$	3	$x=\frac{b}{c-a}=\frac{48}{18-12}=8$. The Number required.

PROBLEM III.

To find a Number, to which if 11 *be added, and* 7 *ſubtracted from the ſame Number,* (viz. *the firſt*) *the Sum of the Addition may be double the Remainder.*

Put 11=*a*, 7=*b*, and call the Number ſought *x*; then by the Queſtion it will ſtand

	thus	1	$x+a=2x-2b$
1	$+2b$	2	$x+a+2b=2x$
2	$-x$	3	$x=a+2b=11+\overline{7\times2}=25$. The Number found.

PROBLEM IV.

To find a Number, to which if it's double (treble, quadruple, &c.) *be added, the Square of the ſame Number may be produc'd.*

CALL the Number ſought x, to which, by the Queſtion, add it's double, *viz.* $2x$, and the Sum is $x+2x$ or $3x$; which Sum, *per Query*, is equal the Square of x, that is, $=x^2$. *Ergo* $3x=x^2$; this Equation divided by x, gives $x=3$. But the Queſtion ſays, if treble the Number ſought, *viz.* x, be added to itſelf, the Sum will alſo be equal to the Square of x; therefore $x+3x$, or $4x=x^2$, and dividing this Equation by x, we ſhall have $x=4$; and again, if we add to x it's Quadruple, the Sum will likewiſe be equal to x^2, conſequently $x+4x=x^2$, that is, $5x=x^2$; which being divided by x, the Queſtion will give $x=5$, *&c.*

Q. E. I.

PROBLEM V.

To find a Number, which if added to it ſelf and the Sum multiplied by the ſame, and the ſame Number ſtill ſubtracted from the Product: And laſtly, the Remainder divided by the ſame, that it may produce 13.

For the Number ſought put x, which added to itſelf, the Sum is $2x$, this Sum multiplied by x, the Product is $2x^2$; from this Product ſubtract x, and the Remainder is $2x^2 - x$: And laſtly, divide this Remainder by x, and the Quotient is $\frac{2x^2 + x}{x}$, but becauſe x is found in both the Terms of the Numerator, the Quotient will be $2x - 1$, which, by the Queſtion, is equal $13 = a$. Therefore,

$$\begin{array}{r r|r|l} & & 1 & 2x - 1 = a \\ 1 & +1 & 2 & 2x = a + 1 \\ 2 & \div 2 & 3 & x = \frac{a + 1}{2} = \frac{13 + 1}{2} = 7. \end{array}$$

Q. E. I.

PROBLEM VI.

To divide the Number 16 into 2 Parts, ſo that the Square of the greater Part may exceed the Square of the leſs by 32.

To ſolve this Propoſition according to *Ward*'s Method, put x the greater Part, and y the leſs; and the Square of the greater Part x is x^2, and the Square of the leſſer is y^2, the Difference of theſe two Squares is x^2-y^2, which is equal to 32 by the Queſtion.

Therefore		1	$x+y=16$
		2	$x^2-y^2=32$
1	$-x$	3	$y=16-x$
3	@ 2	4	$y^2=256-32x+x^2$
2	$+y$	5	$x^2=32+y^2$
5	-32	6	$y^2=x^2-32$
4	$=6$	7	$256-32x+x^2=x^2-32$
7	$-x$	8	$256-32x=-32$
8	$+32$	9	$256=32x-32$
9	$+32$	10	$32x=288$
10	$\div 32$	11	$x=\frac{288}{32}=9$, the greater Part,

which taken from 16, the Remainder is 7, the leſſer Part. *Q. E. I.*

Note, This Method of Exterminating unknown Quantities, is called *Comparative Algebra.*

☞ But the great and incomparable Mathematician, Sir *Iſaac Newton*, has, in his *Univerſal*

verſal Arithmetic, Page 68, moſt wiſely noted, that the Solutions of Queſtions are (for the moſt part) ſo much the more expedite and artificial, by how fewer unknown Quantities you have at firſt. That is, in thoſe Queſtions where two, three, or more unknown Quantities are required to be found, by the Aſſumption of one unknown Quantity only, the reſt, many Times, will be expreſs'd by ſuch and ſuch Laws or Properties of *Algebra* of *Geometry*, as the Query happens to fall under: With ſuch Expreſſions, the Queſtion being ſtated, and the unknown or aſſum'd Quantity being inveſtigated, the other unknown Numbers will be had from the Expreſſions themſelves.

THE better to conceive this Method of Reſolution, the Algebraiſt is deſir'd to obſerve the following

AXIOMS.

1. IF from the Sum of any two Quantities be taken either of them, the Remainder will give the other Quantity.

2. THE Difference of any two Quantities added to the leſs, gives the greater Quantity; and ſubtracted from the greater Quantity, gives the leſſer; or the whole is equal to all the Parts taken together.

3. THE Product of any two Quantities divided by either of them, the Quotient ariſing will give the other Quantity.

4. The Quotient of any two Quantities, being multiplied by the leſſer, the Product is the greater Quantity.

N. B. *That what is here ſaid of ſimple Quantities, the ſame may be underſtood of Squares, Cubes, and Biquadratic Quantities*, &c.

Now, by a right Application of theſe Axioms, many Queſtions in *Algebra*, where 2, 3, 4, *&c.* Quantities are requir'd to be found, may be expeditiouſly ſolv'd, by the Help of one unknown Quantity only. Thus:

Put $16=a$, $32=b$, call the greater Part x, as before, then (by *Axiom* 1.) the leſſer Part will be $a-x$; now, the Square of the greater Part is x^2, and the Square of the leſſer Part is $a^2-2ax+x^2$, which taken from x^2, the Square of the greater Part, the Remainder is $2ax-a^2$, which by the Queſtions is equal to b.

Therefore,	1	$2ax-a^2=b$
1 $+a^2$	2	$2ax=b+a^2$
2 $\div 2a$	3	$x=\dfrac{b+a^2}{2a}=\dfrac{b}{2a}+\dfrac{a}{2}$ $\dfrac{32}{2\times16}+\dfrac{16}{2}=9.$

the greater Part, the ſame as before, whence the leſſer Part is 7. *Q. E. I.*

By comparing this Method with that aforegoing, we ſhall find the very ſame Concluſion is produc'd with quarter the Trouble; for, in that Operation, as there are two unknown Quantities, x and y, concern'd, there muſt be one of them, as y, exterminated before the other can be found; which Trouble of Extermination is entirely avoided by this Method of Subſtitution.

PROBLEM VII.

To divide the Number 36 *into two Parts, ſo that if* 12 *be added to the firſt, and* 6 *to the ſecond, the former may be the double of the latter.*

PUT $36=a$, $12=b$, $6=c$, and call the firſt Part x, then the ſecond Part (by *Axiom* 1.) will be $a-x$; now, if to the firſt Part x be added b, the Sum is $x+b$, and to the ſecond Part $a-x$ be added c, the Sum is $a-x+c$, which is half the Sum $a+b$, *per Query*.

$$\begin{array}{ll|l|l}
\text{Therefore,} & & 1 & x+b=2a-2x+2c \\
1 & +2x & 2 & 3x+b=2a+2c \\
2 & -b & 3 & 3x=2a+cx-b \\
3 & \div\bar{3} & 4 & x=\dfrac{2a+2c-b}{3}=\dfrac{\overline{2\times36}+\overline{2\times6}-12}{3} \\
 & & & =24,
\end{array}$$

the firſt or greateſt Part; which being known, the leſſer Part is $36-24=12$; and theſe two Numbers will anſwer the Conditions of the Queſtion. *Q. E. I.*

PROBLEM

PROBLEM VIII.

Let the Line A B (*of* 70 *Parts*) *be divided any how in* C, [*ſo that* A C *may be* 42, B C 28] *it is required to divide the ſame Line again in another Point:* For Example, *in* D, *ſo that the Rectangle* A D C *may be equal to the Square* D B. *Let the Segment* C D *be enquired,* [*which being obtained,* A D, B D *will be known.*]

PUT $70=a$, $42=b$, $28=c$, and let x repreſent the Segment CD, and AD (by *Axiom* 1 and 2.) will be $b+x$, $DB=c-x$; now multiplying AD by DC, or their Equivalents $b+x$ by x, the Product is $bx+x^2$, which (by the Queſtion) is equal the Square of DB, or it's Equivalent $c-x$, which is $c^2-2cx+x^2$. (*See Fig.* 1.)

Therefore,	1	$bx+x^2=c^2-2cx+x^2$
1 $-x^2$	2	$bx=c^2-2cx$
2 $+2cx$	3	$bx+2cx=c^2$
3 $\div\overline{b+2c}$	4	$x=\dfrac{c^2}{b+2c}=\dfrac{28\times28}{42+56}=8$, the Length

of CD; which being known, gives $AD=50$, and $DB=20$. *Q. E. I.*

The ſame otherwiſe.

LET $x=AD$, whence $a-x=DB$, and $CD=x-b$, as is evident by the Line AB; and multiplying $x-b$ by x, the Product is x^2-bx, which is equal to the Square of $a-x$ or to $a^2-2ax+x^2$ by the Queſtion.

Therefore,	1	$x^2-bx=a^2-2ax+x^2$
1 $-x^2$	2	$-bx=a^2-2ax$
2 $+2ax$	3	$2ax-bx=a^2$
3 $\div 2a-b$	4	$x=\frac{a^2}{2a-b}=\frac{70\times70}{2\times70-42}=50=AD$,

Geometrically thus.

ON the given Segment CB, (by the 46th. *Prop.* of *Euclid*'s firſt Book) deſcribe the Square *cb*BC, and draw the Diagonal C*b*; this done, compleat the Parallelogram *ab*AB, and draw its Diagonal *a*B, which cuts the other Diagonal C*b* in the Point *c*. Now, through the Point *e*, and parallel to *b*B, or *c*C, draw the Line *d*D, and it cuts the Segment CB in D, the Point requir'd. (*See Fig.* 2.)

Demonſtration.

DRAW through the Point *e* the Line *t r s*, parallel to the given Side A B; then (*per* Cor.

to *Prop.* 4. of *Euclid*'s ſecond Book) the Rectangles $reDC$ and $dbSe$ are Squares, therefore eD is $= CD$, and $bS = eS = DB$, conſequently $\square bS = \square eS = \square DB$; but the Square of bS or eS is equal the Square $dbSe$, therefore the Squares $dbSe$ and $\square DB$ are equal; and (by *Prop.* 43 of *Euclid*'s firſt Book) the Square $dbSe$ is equal to the Parallelogram $teAD$, which is equal the Rectangle of AD into D*e*, which is equal to AD into CD, for *e*D and CD are equal; therefore the Rectangle of AD into CD is = the Square of DB. *Q. E. D.*

Or by conſtructing Equation the third, of Operation the firſt, aforegoing, will likewiſe give the Point D Geometrically. But the Limits of this Book being ſo ſmall, obliges me to refer the young Analyſt to *J. C. Sturmius*'s *Matheſis Enucleata*, or to *Harris*'s *Algebra*, where he will meet with ample Satisfaction in this Kind of Geometry.

PROBLEM IX.

Let the Line EF *be divided any how in* G, [*ſo that* EG *may be* 6, *and* GF 4] *it is required to produce this right Line* EF, [*for Example unto* H] *ſo that the Rectangle* EHF *may be equal to the Square* GH; *the Length of* FH *is required.*

Put $10 = a$, $4 = b$, and $x = FH$; then (by *Axiom* 2.) $EH = a + x$, and $GH = b + x$;

multiply $a+x$, by x, and the Product is $ax+x^2$, and the Square of $b+x$ is $b^2+2bx+x^2$, which (by the Queſtion) is equal to $ax+x^2$.

(*See Fig.* 3.)

Therefore,	1	$ax+x^2=b^2+2bx+x^2$
1 $\quad -x$	2	$ax=b^2+2bx$
2 $\quad -2bx$	3	$ax-2bx=b^2$
3 $\div a-2b$	4	$x=\dfrac{b^2}{a-2b}=\dfrac{4\times4}{10-2\times4}=8$; and

thus much muſt the Line E F be produced, to fulfill the Conditions of the Queſtion.

Or thus,

PUT $x=$ EH the whole compounded Line, and (by *Axiom* 2.) $x-a=$ FH, and GH $=x-a+b$; the Rectangle of $x-a$ into x, is x^2-xa, and the Square of $x-a+b$ is $x^2-2ax-2ab+a^2+2bx+b^2$, which is equal to x^2-ax by the Queſtion.

Therefore,	1	$x^2-ax=x^2-2ax-2ab+a^2+2bx+b^2$
1 $\quad \overline{+}$	2	$ax-2bx=a^2-2ab+b^2$
2 $\div a-2b$	3	$x=a+\dfrac{b^2}{a-2b}=18$, from which

take 10, and there remains 8 = FH, as before.

Note. IN the Margin, at the ſecond Step $\overline{+}$ ſignifies *Tranſpoſition*, or the Rectification of the firſt Step by *Addition* and *Subtraction*.

Or likewiſe thus,

LET $x = GH$, and (by *Axiom* 2.) $FH = x - b$, and $EH = x - b + a$; which Terms being ſtated as the Queſtion requires, will ſtand

thus, | 1 | $x^2 = x^2 + ax - 2bx - ab + b^2$
1 ± | 2 | $2bx - ax = b^2 - ab$
2 ÷ $2b - a$ | 3 | $x = \frac{b^2 - ab}{2b - a} = \frac{b}{2} - \frac{ab}{4b - 2a} = 12$, from

which take 4, and the Remainder is $8 = FH$, as in the foregoing Operations.

BECAUSE this Equation $x = \frac{b^2 - ab}{2b - a} = \frac{b}{2} - \frac{ab}{4b - 2a}$ is ſomewhat uncommon, and perhaps may non-plus the Learner, tho' to the more ſkilful in *Algebra* it is moſt evident, it will (I preſume) not be improper to give the Reader its

Explanation.

IN this Fraction $\frac{b^2 - ab}{2b - a}$, ſeeing a is greater than b, it is plain, by the Laws of *Addition*, that the Sum of $b^2 - ab$, the Numerator, is a negative Quantity; and by the ſame Way of Reaſoning, the Denominator $2b-a$ is a negative Quantity alſo: But a negative Quantity divided by a negative Quantity, by the Laws of Diviſion, the Quotient will be Affirmative; therefore if you write 10 for a, and 4 for b, the ſaid Fraction will become $\frac{4\times4-10\times4}{2\times4-10}$, that is, $\frac{-24}{-2}=12=x$. And ſo likewiſe $\frac{b}{2}-\frac{ab}{4b-2a}$, when thrown into Numbers will ſtand thus $\frac{4}{2}-\frac{10\times4}{4\times4-2\times10}$ $=2-\frac{40}{-4}=2+10=12$. Which was to be explained.

PROBLEM X.

A General diſpoſing his Army into a Square Battle, finds he has 284 Soldiers over and above; but increaſing each Side with one Soldier, he wants 25 Soldiers to fill up the Square: How many Soldiers had he?

PUT $284=a$, $25=b$, and let the Number of Soldiers placed in Rank and File be called x; which being ſquar'd is x^2, which is leſs than the Number of Soldiers in the Army by

284 or *a* (*per Query.*) But increaſing each Side with one Soldier, that is, adding one to x the Sum is $x+1$, whoſe Square is $x+2x+1$, which (by the Queſtion) is greater than the whole Army by 25 or b, conſequently taking b from x^2+2x+1 the Remainder is $x^2+2x+1-b$ the whole Army, and adding a to x^2, the Sum is likewiſe equal the whole Army, that is, the Sum of a and x^2 is x^2+a.

Therefore	1	$x^2+a=x^2+2x+1-b$.
1 $-x^2$	2	$a=2x+1-b$
2 $+$	3	$2x=a+b-1$
3 $\div 2$	4	$x=\frac{a+b-1}{2}=\frac{284+25-1}{2}=154$,

the Soldiers in Rank and File, which being ſquar'd, and 284 added to it, that is, add 284 to 23716 the Square of 154, and the Sum is 24000 the Number of Soldiers requir'd.

The ſame otherwiſe.

PUT $x=$ the whole Army, then $x-a$ (by the Queſtion) is a ſquare Number, whoſe Square-root is $\sqrt{x-a}$ the Side of the Square equal the Number of Soldiers in Rank and File, to which add 1, and the Sum is $\sqrt{x-a}+1$, the Square of which is $x-a+2\sqrt{x-a}+1$, and becauſe this Square exceeds the Number of Soldiers by b or 25, ſubtract b from it, and the Remainder is $x-a+2\sqrt{x-a}+1-b$, which (*per Query*) is equal the whole Army.

Therefore,	1	$x-a+2\sqrt{x-a}.+1-b=x.$
1 $\pm$	2	$2\sqrt{x-a}=a+b-1=308=c$
2 $\circledcirc 2$	3	$4x-4a=c^2$
3 $+4a$	4	$4x=c+4a$
4 $\div 4$	5	$x=\frac{c^2+4a}{4}=\frac{308\times 308+4\times 284}{4}$
		$=24000.$ The ſame as above.

Numerically.

THIS numerical Solution depends on the fourth Propoſition of the ſecond Book of *Euclid*. For ſeeing the Number of Soldiers exceeds the leſſer Square by 284, and wants 25 to fill up the greater Square, conſequently 284 + 25 = 309 is equal the double Rectangle of the Side of the leſſer Square into 1 *more* the Square of 1 (by the aboveſaid Propoſition.) But becauſe the Square of 1 is but 1: therefore 308 is the double Rectangle of the Side of the leſſer Square into 1; whence dividing 308 by 2 the Quotient gives 154 the Side of the leſſer Square as before, which ſquar'd produces 23716, to which add 284 the Sum is 24000 the Number of Soldiers in the Army. *Q. E. J.*

PROBLEM XI.

A certain Captain ſends out $\frac{1}{3}$ *of his Soldiers* + 10, *and there remain'd* $\frac{1}{2}$ + 15. *How many Soldiers had he?*

Put $10 = a$, $15 = b$, and $x =$ the Number of Soldiers; then the Number of Soldiers ſent out are $\frac{x}{3} + a$, and thoſe remaining $\frac{x}{2} + b$, which together are equal to x.

Therefore | 1 | $\frac{x}{3} + a + \frac{x}{2} + b = x$.

1 $\times 3$ | 2 | $x + 3a + \frac{3x}{2} + 3b = 3x$

2 $\times 2$ | 3 | $2x + 6a + 3x + 6b = 5x + 6a + 6b = 6x$.

3 $-5x$ | 4 | $x = 6a + 6b = 6 \times 10 + 6 \times 15 = 150$, the Number of Soldiers. *Q. E. J.*

Or thus.

To avoid Fractions put $6x =$ the Number of Soldiers, of which $\frac{1}{3}$ is $2x$, and $\frac{1}{2}$ is $3x$; whence, by the Queſtion, we ſhall have this

Equation, | 1 | $2x + a + 3x + b = 6x$.

Or, | 2 | $5x + a + b = 6x$,

2 $-5x$ | 3 | $x = a + b = 10 + 15 = 25$, which

multiplied by 6 gives 150, the ſame as above.

Numerically:

SEEING $\frac{1}{3}$ of of any Quantity is the ſame as $\frac{2}{6}$, $\frac{1}{2}$ the ſame as $\frac{3}{6}$ of the whole; and that (by the Queſtion) $\frac{1}{3}$ or $\frac{2}{6} + 10$, and $\frac{1}{2}$ or $\frac{3}{6} + 15$ together makes $\frac{5}{6} + 25 =$ the Number of Soldiers; it is evident that 25, the Sum of 10 and 15, is $\frac{1}{6}$ Part of the whole Company. Therefore $25 \times 6 = 150$ the Number of Soldiers required. *W. W. D.*

PROBLEM XII.

There is an Army to which if you add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itſelf, and take away 5000, *the Sum total will be* 100000. *What was the Number of the Army?*

TO avoid Fractions put $24x =$ the Number of the Army, for 24 is a Multiple of 2, 3, and 4, the Denominators of theſe Fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; Now $\frac{1}{2}$ of $24x$ is $12x$, $\frac{1}{3}$ is $8x$, and $\frac{1}{4}$ is $6x$, whence (by the Queſtion) we have | 1 | $24x + 12x + 8x + 6x - 5000 = 100000.$

Or, | 2 | $50x - 5000 = 100000.$

2 $+5000$ | 3 | $50x = 105000$

3 $\div 50$ | 4 | $x = \frac{105000}{50} = 2100$ which multiplied by 24 gives 50400, the Number of the Army. *Q. E. J.*

PROBLEM XIII.

In the Rectangle ABCD, *the Difference of the greater Side* AB, *and of the leſſer Side* BC, *is* 12, *but the Difference of the Squares of the Sides is* 1680. *What are the Sides of the Rectangle* ABCD?

Put $AB - BC = 12 = a$, $AB^2 - BC^2 = 1680 + b$, and $x = BC$ the leſſer Side; and (*per Ax.* 2.) the greater Side $AB = x + a$, whoſe Square is $x^2 + 2ax + a^2$, and the Square of the leſſer Side is x^2; the Difference of theſe two Squares is $2ax + a^2$, which (*per Query*) is equal b. (*See Fig.* 4.)

Therefore | 1 | $2ax + a^2 = b.$

1 $-a^2$ | 2 | $2ax = b - a^2$

2 $\div 2a$ | 3 | $x = \frac{b}{2a} - \frac{a}{2} = \frac{1680}{24} - \frac{12}{2} = 64 = BC,$

hence $AB = 64 + 12 = 76$, the Sides required.

PROB.

PROBLEM XIV.

The Length DE *of the Rectangle* DEFG, *is twice the Breadth* EF; *and the Sum of the Squares of the Length and Breadth is ten Times the Sum of the two Sides* DE, EF. *What are the Sides of the Rectangle* DEFG?

Put the Breadth EF$=x$, and the Length DE will be $= 2x$; the Sum of the Length and Breadth is $2x+x$ or $3x$, and the Sum of their Square is $4x^2+x^2$ or $5x^2$, which (by the Queſtion) is equal 10 Times the Sum of the Sides or $3x$. (*See Fig.* 5.)

Therefore | 1 | $5x^2=30x$.
1 $\div 5x$ | 2 | $x=6=$EF, the leſſer Side, conſequently the greater Side DE is 12, the Side of the Rectangle, as was required.

If the Learner is not yet acquainted with the Doctrine of Proportion, let him peruſe the ſeventh Chapter of the ſecond Part of *Ward*'s *Introduction to Mathematics*; or the fifth Book of *Euclid*'s *Elements of Geometry*, and then he will the eaſierly underſtand the following Solution, and ſeveral others where the Buſineſs of Proportion is required.

PROBLEM

PROBLEM XV.

To find two Numbers in the Proportion of 2 to 3, whoſe Product, if they be multiplied by one another, ſhall be 54.

Put $54 = a$, $x =$ the leſſer Number, and as $2 : 3 :: x : \frac{3x}{2} =$ the greater Number; the Product of x and $\frac{3x}{2}$ is $\frac{3x^2}{2}$, which (by the Queſtion) is equal to a.

Therefore		1	$\frac{3x^2}{2} = a$.
1	$\times\frac{2}{3}$	2	$x^2 = \frac{2a}{3}$
2	ω 2	3	$x = \sqrt{\frac{2a}{3}} = \sqrt{\frac{2 \times 54}{3}} = 6$, the leſſer

Number, and as $2 : 3 :: 6 : 9 =$ the greater Number. Q. E. J.

Least the young Algebraiſt ſhould not readily perceive the Reaſon of multiplying this Equation $\frac{3x^2}{2} = a$, at Step the firſt, by $\frac{2}{3}$, to bring x^2 out of the Fraction, which to the Profecient will be moſt conſpicuous, it is thought proper to give him the following

EXPLANATION.

THAT multiplying $\frac{3x^2}{2}=a$ by $\frac{2}{3}$ brings out $x=\frac{2a}{3}$ at Step the ſecond, will appear very plain if we retain the whole Multiplication; thus, $\frac{3x^2}{2}\times\frac{2}{3}=\frac{2\times3x^2}{3\times2}=\frac{2a}{3}$. For ſince the Co-efficient of x^2 is $\frac{2\times3}{3\times2}$, and becauſe the Numerator and Denominator are the ſame in all Reſpects, they conſequently deſtroy each other, therefore $x^2=\frac{2a}{3}$.

HENCE we have the following general Method for freeing any literal Quantity of a Fractional Co-efficient. Multiply the Quantity by it's Co-efficient inverted, and it is done.

Note, What I mean by an inverted Fraction is this, $\frac{3}{2}$ when inverted becomes $\frac{2}{3}$.

FROM hence alſo appears the Reaſon of multiplying a Fractional Quantity by it's Denominator,

nominator, by only taking away the Denominator. For $\frac{3x-5}{2}$ (at Step 1. Operation 1. of the firſt Propoſition) multiplied by 2 it's Denominator, that is, $\frac{3x-5}{2}\times 2 = \frac{\overline{3x-5}\times 2}{2}$ becomes $3x-5$, at Step 2; becauſe 2 is found both in the Numerator and Denominator, and ſo conſequently vaniſhes.

Numerically.

SEEING that 54 is the Rectangle or Product of the two Quantities required to be found, which Numbers having the ſame Ratio to one another as 2 to 3: Therefore multiply 2 and 3 together, and the Product is 6. Now becauſe (by the twentieth Propoſition of the ſixth Book of *Euclid's Elements*) all ſimilar Planes are in a duplicate Ratio of their like Sides, ſay, as 6 : is to 4 (= the Square of 2 the leaſt Ratio) : : ſo is 54 : to 36, whoſe ſquare Root is 6 = the leſſer Number; and divide 54 by 6, and the Quotient is 9 = the greater Number. *W. W. D.*

PROBLEM

PROBLEM XVI.

To find two Numbers whoſe Ratio is to one another as 4 *to* 5 : *and the Sum of the Squares of both, is* 2624.

Put x = the leſſer Number, and as $4 : 5 :: x : \frac{5x}{4}$ = the greater; the Square of the leſſer Number x is x^2. and the Square of the greater Number is $\frac{25x^2}{16}$, and the Sum of theſe two Squares is $\frac{25x^2}{16}+x^2$, which by the Queſtion is equal $2624 = a$.

Therefore		1	$\frac{25x^2}{16}+x^2=a.$
1	$\overline{\times 16}$	2	$25x^2+16x^2=16a$
2	$\overline{\div 25+16}$	3	$x^2=\frac{16a}{25+16}$
3	$\overline{\omega\ 2}$	4	$x=\sqrt{\frac{16a}{25+16}}=\sqrt{\frac{16\times 2624}{25+16}}=32$

the leſſer Number, which being known, ſay, as $4 : 5 :: 32 : 40$ = the greater Number.

Q. E. J.

Or thus.

TAKE x^2 = the Square of the leſſer Number, from a, and the Remainder is $a-x^2$ = the Square of the greater Part (*per* Note following the *Axioms*), and the Square of the Ratios is 16 and 25; now (by the twenty-ſecond Propoſition of *Euclid's* ſixth Book), ſay,

as	1	$16 : 25 :: x^2 : a-x^2$. And (by the XVIth of the ſame Book)
we have	2	$16a-16x^2=25x^2$
2 $+16x^2$	3	$41x^2=16a$, the very ſame with Step 2, above, only contracted.
3 $\div\overline{41}$	4	$x^2=\frac{16a}{41}$
4 $\overline{\omega 2}$	5	$x=\sqrt{\frac{16a}{41}}=\sqrt{\frac{16\times 2624}{41}}=32$, &c.

Numerically.

THE Squares of the Ratios added together is 41, and the Sum of the Squares of the Numbers required being given = 2624, ſay by Proportion, as 41 : 16 :: 2624 : 1024 = the Square of the leſſer Part, whoſe Root is 32 = the leſſer Number, from 2624 take 1024, and the Remainder is 1600, whoſe ſquare Root is 40, the greater Number. *W. W. R.*

Note, This Numerical Rule is founded on the twenty-ſecond Propoſition of the ſixth, and the eighteenth of the fifth Books of *Euclid*'s *Elements of Geometry.*

PROBLEM XVII.

To find the Side of a Square, whoſe Area is to the Sum of the Sides in a given Ratio, as 45 *to* 12.

PUT $45=a$, $12=b$, and $x=$ the Side of the Square; then $4x$ is the Sum of the Sides, and x^2 the Area of the Square, which are to each other as a to b, by the Propoſition.

Therefore | 1 | $a : b :: x^2 : 4x$.
1 *Ergo* | 2 | $bx^2 = 4ax$ (by 16. *E.* 6.)
2 $\div bx$ | 3 | $x = \frac{4a}{b} = \frac{4\times 45}{12} = 15$ the Side of the Square required.

N. B. At Step the Second, (by 16. *E.* 6.) is to be read thus: By the ſixteenth Propoſition of the ſixth Book of *Euclid*'s Elements.

Numerically.

All Squares are ſimilar Planes; therefore divide the Ratio of the Perimeter or Sum of the Sides by 4, and with that Quotient divide the Ratio of the Area; this laſt Quotient will give the Side of thc Square required. Thus, the Quotient of 12 divided by 4 is 3, and 45 divided by 3 quotes 15 = the Side of the Square as above.

The Reaſon of this Arithmetical Solution, is this, that that Number, that expreſſes the Ratio of any Quantity, whether a Line, Superficies, or Solid, hath the ſame Properties as that Quantity, of which it is the Ratio.

PROBLEM XVIII.

To find the Side of a Cube, whoſe Superficies is to the Solidity in a given Ratio, as 6 *to* 11.

Put $6=a$, $11=b$, and $x=$ the Side of the Cube; then the Solidity will be x^3, and the Superficies $6x^2$. Now by the Queſtion we ſhall

have | 1 | $a : b :: 6x^2 : x^3$.
1 *Ergo* | 2 | $ax^3=6bx^2$
2 $\div ax^2$ | 3 | $x=\frac{6b}{a}=\frac{6\times 11}{6}=11$ the Side of the Cube required.

Numerically

Numerically.

All ſimilar Solids are to each other as the Cubes of their like Sides (by the thirty-third Propoſition of *Euclid*'s eleventh Book), whence divide the Ratio of the Solidity of the Cube, by $\frac{1}{6}$ Part of the Ratio of the Superficies, and the Quotient is the Side of the Cube required. Thus, $\frac{1}{6}$ Part of 6 is 1, and 11 divided by 1 is 11 = the Side of the Cube as above. Or ſuppoſe the Proportion of the Superficies and Solidity had been as 243 to 2916, the Side of the Cube would then be 72; for a ſixth Part of 243 is 40.5, and 2916 divided by 40.5 the Quotient is 72. *W. W. D.*

PROBLEM XIX.

A certain Man hires a Labourer, on this Condition, that for every Day he work'd he ſhould receive 12 *Pence, but for every Day he was idle, he ſhould be mulcted* 8 *Pence. When* 390 *Days were paſt, nether of them were indebted one to another. How many Days did he work; and how many was he idle?*

Put 390 = a, 12 = b, 8 = c, and x = the Days he was idle; then (*per Ax.* 1.) the Days he work'd will be $a-x$. Now ſeeing the 8 Pences diſcounted for the Days he play'd was equal the 12 Pences he was to receive for every Day he work'd, we ſhall

$$\begin{array}{l|l|l} \text{have} & 1 & cx = ab - bx. \text{ For } c : b :: a - x : x. \\ 1 \quad + bx & 2 & bx + cx = ab \\ 2 \quad \div b + c & 3 & x = \dfrac{ab}{b+c} = \dfrac{390\times 12}{12+8} = 234 \end{array}$$

the Days he were idle, which taken from 390 leaves 156 the Days he work'd.

Numerically.

As by the Queſtion 8 times the Number of Days the Labourer was idle, are equal 12 times the Number of Days he work'd; there is nothing more required, then to divide 390 Days into two ſuch Parts, that 8 times the one ſhall be equal 12 times the other, and this Diviſion is done reciprocally (by virtue of the fourteenth Propoſition of *Euclid*'s ſixth Book.) Thus: As 12 + 8 (=20) : 12 :: 390 : 234 = the Days idle; and as 12 + 8 (= 20) : 8 :: 390 : 156 = the Days he work'd, the ſame as before, for $156\times12 = 1872 = 234\times8$.

W. W. D.

PROBLEM

PROBLEM XX.

A certain Gentleman hires a Servant, and promiſes him 24 Pounds yearly Wages, together with a Cloak. At 8 Months End the Servant obtains leave to go away, and inſtead of his Wages receives a Cloak + 13 Pounds. How much did the Cloak coſt?

To ſolve this Problem put 12, the Months in a Year, $= a$, and 8 Months he ſerved $= b$, let the Price of the Cloak be called x; likewiſe put $24 = c$, and $13 = d$. Then $c + x$ is his yearly Wages, and $x + d =$ the Wages he received at 8 Months End (by the Queſtion), therefore ſay,

as	1	$c + x : a :: x + d : b$.
1 *Ergo*	2	$ax + ad = bc + bx$
2 $-bx$	3	$ax - bx + ad = bc$
3 $-ad$	4	$ax - bx = bc - ad$
4 $\div a - b$	5	$x = \frac{bc - ad}{a - b} = \frac{8 \times 24 - 12 \times 13}{12 - 8} = 9$

Pounds the Price of the Cloak. *Q. E. J.*

Numerically.

To ſolve this Propoſition Arithmetically, reaſon thus; 8 Months is $\frac{8}{12}$ or $\frac{2}{3}$ of a Year;

therefore at the End of 8 Months, at the Rate of 24 Pounds and a Cloak *per* Year, the Servant muſt receive $\frac{2}{3}$ of 24 Pounds, together with $\frac{2}{3}$ of the Cloak for 8 Months Service; but becauſe he received the whole Cloak, + 13 Pounds, inſtead of $\frac{2}{3}$ of his Year's Wages, which is 16 Pounds and $\frac{2}{3}$ of the Cloak, therefore take 13 from 16, and the Remainder is 3 Pounds, equivalent to $\frac{1}{3}$ Part of the Cloak; conſequently the Cloak coſt 9 Pounds.

W. W. D.

PROBLEM XXI.

A Perſon being ask'd how old he was, anſwered, If I quadruple $\frac{2}{3}$ *of my Years, and add* $\frac{1}{2}$ *of them* + 50 *to the Product; the Sum will be ſo much above* 100, *as the Number of my Years is now below* 100.

To anſwer this or any Queſtion of the like Nature, the Learner ought to be acquainted with Arithmetical Progreſſion; with which if he be unacquainted, he may conſult the ſixth Chapter of the firſt or ſecond Part of *Ward*'s *Introduction to the Mathematics*

To avoid Fraction, put $6x =$ his Age, of which two thirds is $4x$, and the Quadruple of $4x$ is $16x$, to $16x$ add $3x$ ($= \frac{1}{2}$ his Age) $+ 50$, and the Sum is $19x+50$, which (by the Queſtion) as much exceeds 100, as $6x$ (= his Age) wants of 100. Whence $6x$, 100, and $19x+50$ are a Series of Terms in Arithmetical Progreſſion, and will ſtand

	Thus	1	$6x$. 100. $19x+50$.
	Ergo	2	$25x+50=200$ (*per Lemma* 1. *chap. vi. Part I.* or *Sect.* 1. *chap. vi. Part II.* of *Ward*'s *Introduction*, &c.
2	-50	3	$25x=150$
3	$\div 25$	4	$x=6$, therefore $6x=36$ his Age

as was required.

This Queſtion may alſo be ſtated otherwiſe. Thus, for the Difference between 100 and $6x$ being the ſame as between $19x+50$ and and 100 (*per Query*), that is $100-6x$, and $19x+50-100$ are equal.

	Therefore	1	$100-6x=19x+50-100$.
1	$+6x$	2	$25x+50-100=100$
2	$-50+100$	3	$25x=150$
3	$\div 25$	4	$x=6$, *&c.* the ſame as before.

PROBLEM XXII.

One being aſked what Hour of the Day it was, anſwered, The Day at this Time is 16 *Hours long; if now* ½ *of the Hours paſt be added to* ⅔ *of the Remainder,* you *will have the Hour deſir'd, reckoning from Sun-riſing.*

Put $16 = a$, $x =$ the Hours paſt, and (*per Ax.* 1.) $a - x$ is the Hours remaining; ⅔ of which is $\frac{2a-2x}{3}$, and the half of x is $\frac{x}{2}$, whoſe Sum, *viz.* $\frac{x}{2} + \frac{2a-2x}{3}$ is equal the Hours paſt (by the Queſtion).

Therefore 1 | $\frac{x}{2} + \frac{2a-2x}{3} = x.$

1 $\times 2$ 2 | $x + \frac{4a-4x}{3} = 2x$

2 $\times 3$ 3 | $3x + 4a - 4x = 6x$

3 $-3x+4x$ 4 | $7x = 4a$

4 $\div 7$ 5 | $x = \frac{4a}{7} = \frac{4 \times 16}{7} = 9\frac{1}{7}$ the Hour of the Day, which is $\frac{1}{7}$ of an Hour paſt One o'Clock, reckoning from Sun-riſe; and may

be proved thus: $\frac{112}{7}$ (= the whole Day, or 16 Hours) — $\frac{64}{7} = \frac{48}{7}$ = the Hours remaining, $\frac{2}{3}$ of which is $= \frac{96}{21}$, and $\frac{64}{14}$ is = half the Hours paſt: but theſe two Fractions $\frac{96}{27}$ and $\frac{64}{14}$ are = to each other, for as 96 : 21 :: 64 : 14. *Ergo* 96×14=64×21. *W. W. P.*

Numerically.

BECAUSE $\frac{1}{2}$ the Hours paſt, and $\frac{2}{3}$ of thoſe remaining in one Sum, are equal the ſaid Hours paſt; it is plain that $\frac{2}{3}$ of the Hours remaining and $\frac{1}{2}$ the Hours paſt are equal: Therefore $\frac{4}{3}$ of the Hours of the Day remaining is equal the preſent Time of the Day or Hours paſt; conſequently the Hours of the Day paſt and thoſe remaining, are to one another in the Ratio $\frac{4}{3}$ to $\frac{3}{3}$, or as 1 $\frac{1}{3}$ to 1. Now the Sum of theſe Ratios is $\frac{7}{3}$, and to divide 16 into two Parts that ſhall be to each other

$$\text{as } \tfrac{4}{3} \text{ to } \tfrac{3}{3}, \Big\} \text{ ſay, } \Big\{ \tfrac{7}{3} : \begin{matrix} \frac{4}{3} \\ \frac{3}{3} \end{matrix} :: 16 : \begin{matrix} 9\,\frac{3}{21} \text{ or } 9\frac{1}{7}. \\ 6\,\frac{18}{21}: \end{matrix}$$

the ſame as by the foregoing Operation.

W. W. D.

PROB.

PROBLEM XXIII.

From Noremberg *to* Rome *are* 140 *Miles: A Traveller ſets out at the ſame Time from each of the two Cities, one goes* 8 *Miles a Day, the other* 6, *In how many Days from their firſt ſetting out will they meet one another, and how many Miles will each of them go?*

PUT $x =$ the Number of Days they travel before they meet; then $8x$ is the Number of Miles he goes that travels 8 Miles a Day, and $6x$ is the Number of Miles the other goes that travels 6 Miles; and the Sum of $8x$ and $6x$ is $14x$: which, by the Queſtion, is equal 140 Miles, the Diſtance between *Noremberg* and *Rome*. Therefore $14x = 140$, which Equation divide by 14, we ſhall have $x = 10$ the Number of Days each travelled before they met, whence one travelled 80 Miles and the other 60.

BUT the Number of Days they travelled before they met each other may be found by Diviſion only; for ſeeing they both travel the ſame Number of Days, and that each Day's Journey of both is 14 Miles. Therefore divide 140 by 14 and the Quotient gives 10 the Number of Days as before. Moreover, becauſe every Day's Journey is the ſame, therefore the Place where they meet, divides the Diſtance between *Noremberg* and *Rome* into two ſuch Parts, that are to one another, as the Number of Miles each travels a Day. That is, as 8 to 6, or as 4 to 3.

Whence, as $\left\{ 8+6\,(=14) : \begin{matrix} 8 \\ 6 \end{matrix} :: 140 : \begin{matrix} 80. \\ 60. \end{matrix} \right.$

the ſame as above. *W. W. D.*

PROBLEM XXIV.

A certain Meſſenger goes 6 *Miles every Day:* 8 *Days after, another follows him, and he goes* 10 *Miles a Day. In how many Days will he come up to the firſt?*

PUT $6=a$, $10=b$, $8=c$, and $x=$ the Days he travelled that goes 10 Miles a Day, and the Number of Days the other travelled will be $c+x$: Now the Number of Days each travelled, multiplied by the Number of Miles each Meſſenger goes every Day, gives the Miles both travels ſeparately, which are equal, becauſe they both began their Journey from the ſame Place, and ended it at their overtaking each other; whence $ac+ax$ is the Miles the firſt Meſſenger travelled, and bx the Number of Miles he travels that goes 10 Miles a Day.

Therefore | 1 | $bx = ac + ax$.
1 $-ax$ | 2 | $bx - ax = ac$
2 $\div b - a$ | 3 | $x = \frac{ac}{b-a} = \frac{6 \times 8}{10-6} = 12$, the Days he travelled that goes 10 Miles a Day, and in ſo many Days he came up to the firſt Meſſenger.

Q. E. J.

Or thus.

The firſt Meſſenger had gone 48 Miles before the ſecond Meſſenger ſet out; but becauſe the ſecond Meſſenger gains 4 Miles every Day of the firſt; therefore 48 divided by 4, the Quotient is 12 the Number of the Days as before.

W. W. D.

PROBLEM XXV.

A certain Meſſenger goes 6 *Miles a Day: And after he has gone* 56 *Miles, another follows him who goes* 8 *Miles a Day. In how many Days will he come up to him?*

The foregoing Solution being rightly underſtood, it will be needleſs to give this an Algebraical Inveſtigation; and indeed the

Arithmetical Solution is the very ſame as that aforegoing, for divide 56 by 2 ($=8-6$) and the Quotient is 28, the Number of Days in which the ſecond Meſſenger came up to the firſt. *Q. E. J.*

If the Learner deſire to ſee more of this Kind of Problems, let him peruſe the fifth Propoſition (*Page* 72.) of the Arithmetical Queſtions, in Sir *Iſaac Newton*'s *Univerſal Arithmetic*, with its Caſes and Examples.

PROBLEM XXVI.

One bought 3 *Books, whoſe Prices were in Proportion as* 12, 5, 1: *If the Price of the firſt be doubled, of the ſecond trebled, of the third quadrupled; the Sum of theſe Products will as much exceed* 10 *Crowns, as the Sum of the Prices of the greateſt and middle is below* 5. *How much did the ſaid Books coſt?*

Put $x=$ the Price of the firſt Book, and

as $12 : x :: 5 : \frac{5x}{12} =$ the Price of the ſecond;

and again, as $12 : x :: 1 : \frac{x}{12} =$ the Price of the third: Now double the Price of the firſt

Book is $2x$, treble the ſecond $=\frac{5x}{4}$, and quadruple the Price of the third is $\frac{x}{3}$; the Sum of theſe Products is $2x+\frac{5x}{4}+\frac{x}{3}$, which as much exceeds 10 Crowns, as $x+\frac{5x}{12}$ (= the Sum of the greateſt and middle Prices) is below 5. (*per Query.*)

$$\begin{array}{l|l|l} \text{Therefore} & 1 & 2x+\frac{5x}{4}+\frac{x}{3}-10=5-x-\frac{5x}{12}. \\ 1\times 12 & 2 & 43x-120=60-17x \\ 2+17x & 3 & 60x-120=60 \\ 3+120 & 4 & 60x=180 \\ 4\div 60 & 5 & x=3, \end{array}$$

the Number of Crowns the firſt Book coſt = 15 Shilling, and, as 12 : 15 :: 5 : 6 ¼ Shillings, the Price of the ſecond; and again, as 12 : 15 :: 1 : 1 ¼ Shilling, the Price of the third Book. *Q. E. J.*

PROBLEM

PROBLEM XXVII.

Suppoſe the Number 50 *were to be divided into two Parts, ſo that the greater Part being divided by* 7, *and the leſs multiplied by* 3, *the Sum of this Product and the former Quotient may make the ſame Number propoſed, which was* 50.

PUT $50 = a$, $7 = b$, $3 = c$ and $x =$ the greater Part; then the leſs (*per Ax.* 1.) will be $a - x$: again, $\frac{x}{b}$ is a 7th Part of x, and $ac - cx$ is three Times $a - x$; the Sum of this Quotient and Product is $\frac{x}{b} + ac - cx$, which (by the Queſtion) is equal a.

Therefore	1	$\frac{x}{b} + ac - cx = a.$
1 $\times b$	2	$x + abc - bcx = ab$
2 $\underset{=}{+}$	3	$bcx - x = abc - ab$
3 $\div bc - 1$	4	$x = \frac{abc - ab}{bc - 1} = 35$, the greater

Part; conſequently the leſſer is 15. *Q. E. J.*

PROBLEM XXVIII.

Let the Number 20 be divided into two Parts, ſo that the Square of the leſs Part, being taken out of the Square of the greater, may leave the very Number propoſed, which was 20 (or may leave the double, treble, &c. *of the Number propoſed.)*

PUT 20 $= a$, $x =$ the leſſer Part, and (*per Ax.* 1.) the greater Part will be $= a - x$; the Square of the greater Part is $a^2 - 2ax + x^2$, and the Square of the leſſer x^2, which taken out of the Square of the greater Part, leaves $a^2 - 2ax$, this Remainder (by Propoſition) is equal to a, or $2a$, or $3a$, *&c.*

Therefore	1	$a^2 - 2ax = a.$
$1 + 2ax$	2	$a + 2ax = a^2$
$2 - a$	3	$2ax = a^2 - a$
$3 \div 2a$	4	$x = \frac{a^2 - a}{2a} = \frac{a-1}{2} = \frac{20-1}{2} = 9.5$

the leſſer Part, and $20 - 9.5 = 10.5$ the greater. *Q. E. I.*

AND becauſe $a^2 - 2ax$ may be equal $2a$, we ſhall have this new

$$\begin{array}{l|c|l} \text{Equation} & 1 & a^2-2ax=2a. \\ 1 \quad + & 2 & 2a=a^2-2a \\ 2 \quad \div 2a & 3 & x=\frac{a^2-2a}{2a}=\frac{a}{2}-1=9 \end{array}$$

the leſſer Part, and the greater is $=20-9=11$.

And again, a^2-2ax being made equal to $3a$, we ſhall have

$$\begin{array}{l|c|l} \text{have} & 1 & a^2-2ax=3a. \\ 1 \quad + & 2 & 2ax=a^2-3a \\ 2 \quad \div 2a & 3 & x=\frac{a^2-3a}{2a}=\frac{a}{2}-\frac{3}{2}=\frac{20}{2}-\frac{3}{2}=8.5 \end{array}$$

the leſſer Part, and $20-8.5=11.5$ is the greater, and ſo on. *W. W. D.*

PROBLEM XXIX.

If a Man gains 30 Crowns a Week, how much muſt he ſpend a Week to have 500 Crowns, together with the Expence of 4 Weeks remaining at the Year's End?

Supposing the Year to conſiſt of 52 Weeks, he will gain $52\times30=1560$ Crowns every Year: Put $x=$ his weekly Expence, and $52x$ Crowns is what he lays out yearly, which taken from 1560 the Remainder is $1560-52x$, which, by the Queſtion, is equal to $500+4x$.

Therefore

Therefore	1	$1560 - 52x = 500 + 4x.$
1 $\overset{+}{=}$	2	$56x = 1060$
2 $\div 56$	3	$x = \frac{1060}{56} = 18\frac{13}{14}$, the Crowns he

ſpends *per* Week, equal to 4*l.* 7*s.* 1*d.* $\frac{5}{7}$.

Q. E. J.

PROBLEM XXX.

A Labourer, after 40 Weeks in which he had been at work, lays up 28 Crowns — the Pay of three Weeks; and finds that he had expended 36 Crowns + the Pay of eleven Weeks. What Pay did he receive a Week?

Put x = his weekly Pay, and $40x$ is the Number of Crowns he received for 40 Weeks Labour; in which Time he laid up $28 - 3x$ Crowns, and expended $36 + 11x$ Crowns, which two Sums together make $8x + 64$ Crowns, equal to $40x$.

Therefore	1	$8x + 64 = 40x.$
1 $-8x$	2	$32x = 64$
2 $\div 32$	3	$x = \frac{64}{32} = 2.$ Whence he received

2 Crowns or 10 Shilling a Week. *W. W. R.*

PROBLEM XXXI.

In the Rectangle ABC, *is given the Baſis* AB = 9, *and the Difference of the other Sides, that is, the Segment* BD = 3. *Required the Sides* AC, BC?

Let 9 = a, 3 = b, and x = CA; and becauſe CA and CD are equal (*See Fig.* 6.) BC $= b + x$. Now (by the 47th Propoſition of *Euclid*'s firſt Book) $BC^2 = AC^2 + AB^2$, but $BC^2 = b^2 + 2bx + x^2$, $AC^2 = x^2$, and $AB^2 = a^2$.

Therefore | 1 | $b^2 + 2bx + x^2 = x^2 + a^2$.
1 $-x^2$ | 2 | $b^2 + 2bx = a^2$
2 $-b^2$ | 3 | $2bx = a^2 - b^2$
3 $\div 2b$ | 4 | $x = \frac{a^2 - b^2}{2b} = \frac{9 \times 9 - 3 \times 3}{2 \times 3} = 12$,

the Side CA, and 12 + 3 = 15 the Length of the Side CB. *Q. E. J.*

Or thus.

The ſame Geometrical Problems may be ſolved by different Principals of Geometry, as well as Arithmetical Queſtions by different Laws of Arithmetic. For by drawing the Circle ADE from C as Center, with the Radius CA, and continuing the Side BC to E;

then will BE be $= b + 2x$, and (by the 36th Propoſition of *Euclid*'s third Book) it will be,

As,	1	$BE : BA :: BA : BD.$ Or rather $BE \times BD = BA^2$.
That is,	2	$b + 2x : a :: a : b.$
2 *Ergo*	3	$b^2 + 2bx = a^2$
3 $- b^2$	4	$2bx = a^2 - b^2$
4 $\div 2b$	5	$x = \frac{a^2 - b^2}{2b} = \frac{9 \times 9 - 3 \times 3}{2 \times 3} = 12$

FROM the Solutions to the foregoing Problem, the Analyſt may obſerve, that in anſwering Queſtions of this Nature, how well he ought to be acquainted with the Principles of Geometry. For when a Queſtion of this Kind is propoſed, it will be impoſſible for him to give an Anſwer to it, unleſs he thoroughly underſtands the Nature of the Figure the Quere is propoſed in, and the different Properties that may ariſe from the Application of new Lines not firſt concerned in the Diagram. For as Propoſitions relating to abſtract Quantities, are ſolved by the Laws of Arithmetic; ſo we muſt have recourſe to the Laws and Properties of Geometrical Figures, to anſwer Queſtions in Geometry. And how the ſaid Properties may be employed in bringing Geometrical Problems to Equations, the moſt illuſtrious Geometer Sir ISAAC NEWTON, in his *Uni-*

verſal Arithmetic, from *Page* 86 to 101, hath largely ſpecified by a Variety of Examples; which ought to be diligently and moſt attentively peruſed, and well underſtood by every one that deſigns to be a Proficient in this Branch of Mathematical Learning.

THE thirty-ſixth Propoſition of *Euclid*'s third Book being well underſtood, it will be eaſy to inveſtigate the Side CA independent of Algebra, by reaſoning thus; Seeing $BE \times BD = BA^2$. And becauſe BD and BA are given, BE will be known alſo; and taking BD from BE, the Remainder DE will be given likewiſe; but (by the *Fig*) DE is $= 2CA$. Therefore $CA = \frac{DE}{2} = DC$: Whence, dividing 81 ($=BA^2$) by 3 ($=BD$) the Quotient gives $BE = 27$, from which take 3, and there remains $DE = 24 = 2CA$, conſequently $CA = 12$. Having thus found CA, add to it $BD = 3$, and the Sum is $BC = 15$ the other Side of the Triangle ABC. Which was required.

Geometrically.

BECAUSE the Side CA, and the Segment BD are together in one Sum equal to the Side BC, and that the Angle BAC is a right one by the Propoſition: Therefore draw A*b* ($=BD$) perpendicular to BA, and join B*b* with a right Line (*See Fig.* 7.) which biſect in *a*; at this Point *a* erect the Perpendicular *a*C until it concurs with *b*A produced to C; laſtly, draw the Line BC, and BAC is the Triangle that was to be conſtructed.

Demonſtration.

THE Angles at *a* being right, and the Baſis of the Triangles B*a*C and *ba*C, *viz.* B*a* and *ba*=, and alſo C*a* common to both, the Sides CB and C*b* (by the fourth Propoſition of *Euclid*'s firſt Book) are equal, and ſo (by Definition 21 of the ſame Book) the Triangle BC*b* is *Iſoſceles.* With CA Radius, deſcribe the Arch AD, ſo ſhall BD be equal *b*A. Therefore, *&c.*

Q. E. D.

PROBLEM XXXIII.

In the Rectangle Triangle ABC, *is given the Baſis* AB $= 5$, *and the Sum of the other Sides* AC$+$BC$=25$. *Required, the Sides* AC, BC *ſeverally?*

PUT $25=a$, $5=b$, and $x=$ CA; then (*per Ax.* 1.) CB$=a-x$. And becauſe the Angle CAB is right, we have (by the forty ſeventh Propoſition of *Euclid*'s firſt Book) $BC^2 = CA^2 + BA^2$; but $BC^2 = a^2 - 2ax + x^2$, $CA^2 = x^2$, and $BA^2 = b^2$. (*See Fig.* 8.)

Therefore 1 $a^2 - 2ax + x^2 = b^2 + x^2$.
1 $-x^2$ 2 $a^2 - 2ax = b^2$
2 $+$ 3 $2ax = a^2 - b^2$
3 $\div 2a$ 4 $x = \frac{a^2 - b^2}{2a} = \frac{a}{2} - \frac{b^2}{2a} = 12 = CA$,

and $25 - 12 = 13 = CB$. *W. W. R.*

Or thus.

PUT $x = BC$, and (*per Fig.*) AC will be $= a - x$; having thus expreſſed every Side of the Triangle ABC, we ſhall have (by the forty-ſeventh Propoſition of *Euclid*'s firſt Book) $BC^2 = AC^2 + AB^2$.

That is, 1 $x^2 = a^2 - 2ax + x^2 + b^2$.
1 $-2x$ 2 $0 = a^2 - 2ax + b^2$
2 $+ 2ax$ 3 $2ax = a^2 + b^2$
3 $\div 2a$ 4 $x = \frac{a}{2} + \frac{b^2}{2a} = \frac{25}{2} + \frac{5 \times 5}{2 \times 25} = 13$

$= BC$, and $25 - 13 = 12 = AC$. The ſame as before.

Note, The Sides AC and BC may be found Numerically, Thus: Compleat the Circle ADE, and (by the thirty-ſixth Propoſition of *Euclid*'s third Book) the Rectangle of BE into the Segment BD is = to the Square of AB; Therefore BD is equal to the Square of AB divided by BE ($= BC + AC = 25$.) That is, BD

$= \frac{5 \times 5}{25} = 1$, and (*per Fig.*) $\frac{25-1}{2} = 12 = AC$, hence $25 - 12 = 13 = BC$.

The ſame Geometrically.

THUS, Draw the Line AB = 5, from any Scale of equal Parts; at right Angles to which, draw the Line A*b* = 25, the given Sum of the Sides AC and BC. This done join B*b* (*See Fig.* 9) with a right Line; which divide into two equal Parts in the Point *a*; from *a* erect the Perpendicular *a*C, and having drawn the Line BC, the Triangle ABC is that which was required to be conſtructed.

Demonſtration.

THE Line *a*C being perpendicular to B*b*, and cuts it into two equal Parts, the Triangle BC*b* (by the fifth Propoſition of *Euclid*'s firſt Book) is *Iſoſceles*. *Ergo* BC = *b*C, conſequently AC + BD = AC + *b*C. *Q. E. D.*

HAVING thus far exerciſed the young Analyſt in ſimple Equations only, it may not be improper to ſhew him what quadratic adfected Equations are, and how ſuch Equations are brought to a Solution: And for the better Underſtanding of which, it is requiſite he ſhould know how they are firſt produced, and this he will eaſily perceive by the following Examples.

EXAMPLE I.

IF inſtead of what is given and required to be found in the foregoing Problem, there had been given the Sum of the Legs forming the right Angle (*See Fig.* 8.) *Viz.* $AB + AC = a$, and the Hypothenuſe or longeſt Side $BC = b$; to find the ſaid Legs ſeparately. Then for the Leg AB put x, and (by *Ax.* 1.) the other Leg AC will be equal $a - x$; having thus expreſs'd every Side of the Triangle ABC, we ſhall have, by Virtue of the 47th Propoſition of *Euclid*'s firſt Book, this

Equation,	1	$BC^2 = AB^2 + AC^2$
That is,	2	$b^2 = x^2 + a^2 - 2ax + x^2.$
1 $\pm$	3	$2x^2 - 2ax = b^2 - a^2$
3 $\div \overline{2}$	4	$x^2 - ax = \frac{b^2 - a^2}{2} = -c.$
4 C $\square$	5	$x^2 - ax + \frac{a^2}{4} = \frac{a^2}{4} - c$
5 ww $\overline{2}$	6	$x - \frac{a}{2} = \sqrt{\frac{a^2}{4} - c}.$
6 $+\frac{a}{2}$	7	$x = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - c}.$

Becauſe a^2 is greater than b^2, their Difference will be a Negative Quantity.

Having thus found the Leg AB, the other Leg AC is given from what is ſhewn above.

SEEING therefore at the fourth Step of the foregoing Operation, there is produced this Equation $x^2 - ax = \frac{b^2 - a^2}{2}$; which becauſe it is conſtituted of two different Powers of the unknown Quantity, *viz.* x^2 and x, whoſe Indices are double to each other, is called a quadratic adfected Equation. Which with reſpect had to the Signs + and − admits of three Varieties or Caſes, *viz.*

$$\left.\begin{array}{l|l} 1 & x^2 + mx = n. \\ 2 & x^2 - mx = n. \\ 3 & mx - x^2 = n. \end{array}\right\} \text{ or } \left\{\begin{array}{l} x^4 + mx^2 = n. \\ x^4 - mx^2 = n. \\ mx^2 - x^4 = n. \end{array}\right\} \text{\&c.}$$

N. B. That the Forms of the above Equations might be rendered the more Univerſal, I put m for any Number that may happen to be the Co-efficient of the unknown Quantity x, and n for the abſolute Number, as $\frac{b^2 - a^2}{2}$, and the like.

And for the Solution of ſuch Equations, the Learner muſt obſerve, that the Method took it's firſt Original from this Conſideration. That ſeeing on one Side of the Equation in every Caſe, there is the Square of x, and the Rectangle of x drawn into the given Quantity m; or what comes to the ſame Purpoſe, the Square of x, and the double Rectangle of x drawn into $\frac{m}{2}$; it is evident (from the 4th Propoſition of *Euclid*'s ſecond Book) that $x^2 + mx$ in the firſt Caſe, $x^2 - mx$ in the ſecond, and $mx - x$ in the third, wants only the Square of $\frac{m}{2}$ that is $\frac{m^2}{4}$ to make the ſaid Quantities compleat Squares; for the Square of $x + \frac{m}{2}$ is $= x^2 + mx + \frac{m^2}{4}$, of $x - \frac{m}{2}$ is $= x^2 - mx + \frac{m^2}{4}$, and of $x - \frac{m}{2}$ is $= x^2 - mx + \frac{m^2}{4}$, which tho' in every Reſpect it is the ſame in Form to that next before it, yet it being in the third Caſe is of a different Quality, becauſe equal to a negative Quantity, as ſhall be ſhewn by and by. Whence proceeds this general Rule for compleating the Square in all quadratic adfected Equation whatſoever.

R U L E.

ADD the Square of $\frac{1}{2}$ the Co-efficient of the unknown Quantity, to both Sides the Equation, and the Square will be compleat. See the following Operations.

$$\begin{array}{l|l|l|l|} 1 & x^2+mx+\frac{m^2}{4}=n+\frac{m^2}{4}. & & x^4+mx^2+\frac{m^2}{4}=n+\frac{m^2}{4}. \\ 2 & x^2-mx+\frac{m^2}{4}=n+\frac{m^2}{4}. & \text{or} & x^4-mx^2+\frac{m^2}{4}=n+\frac{m^2}{4}. \\ 3 & x^2-mx+\frac{m^2}{4}=\frac{m^2}{4}-n. & & x^4-mx^2+\frac{m^2}{4}=\frac{m^2}{4}-n. \end{array}$$

and ſo on. Hence, by extracting the ſquare Root, we ſhall have the following Equations.

$$\begin{array}{r|l|c|l}
1 & x+\frac{m}{2}=\sqrt{n+\frac{m^2}{4}}. & & x^2+\frac{m}{2}=\sqrt{n+\frac{m^2}{4}}. \\
2 & x-\frac{m}{2}=\sqrt{n+\frac{m^2}{4}}. & \text{or} & x^2-\frac{m}{2}=\sqrt{n+\frac{m^2}{4}}.\quad \text{\&c.} \\
3 & x-\frac{m}{2}=\sqrt{\frac{m^2}{4}-n}. & & x^2-\frac{m}{2}=\sqrt{\frac{m^2}{4}-n}.
\end{array}$$

And by tranſpoſing $\frac{m}{2}$ to the other Side, every Equation will give the Value of x.

$$\begin{array}{r|l|c|l}
1 & x=-\frac{m}{2}+\sqrt{n+\frac{m^2}{4}}. & & x^2=-\frac{m}{2}+\sqrt{n+\frac{m^2}{4}}. \\
2 & x=\frac{m}{2}+\sqrt{n+\frac{m^2}{4}}. & \text{or} & x^2=\frac{m}{2}+\sqrt{n+\frac{m^2}{4}}. \\
3 & x=\frac{m}{2}\pm\sqrt{\frac{m^2}{4}-n}. & & x^2=\frac{m}{2}\pm\sqrt{\frac{m^2}{4}-n}.
\end{array}$$

Except in thoſe Equations on the right Hand, which muſt have the Root again extracted. Thus,

$$\begin{array}{|l}
x=\sqrt{-\frac{m}{2}+\sqrt{n+\frac{m^2}{4}}}.. \\
x=\sqrt{\frac{m}{2}+\sqrt{n+\frac{m^2}{4}}}.. \\
x=\sqrt{\frac{m}{2}\pm\sqrt{\frac{m^2}{4}-n}}..
\end{array}$$

N. B. In the third Caſe, or this Equation $mx - x^2 = n$, before the Square can be compleated, the Signs of every Term in the Equation muſt be firſt changed; which being done, will ſtand thus $x^2 - mx = - n$; and then by compleating the Square, it will become $x^2 - mx + \frac{m^2}{4} = \frac{m^2}{4} - n$. as is ſpecified in the foregoing Opererations.

TO what is here ſaid concerning quadratic adfected Equations, much might be added, but I ſhall content myſelf by inſerting only theſe Obſervations following. And if the induſtrious Reader deſire to ſee more of this Doctrine, he is referred to *Ward*'s *Introduction*, Chap. viii. Part II. or to *John Parſon*'s *Clavis Arithmeticæ*, Chap. xv. where he will meet with Plenty of Examples of this Kind.

FIRST then, It may be obſerved that x, the unknown Quantity, in the firſt Caſe, may be either greater or leſs than its Co-efficient, becauſe all the Terms in the Equation are affirmative. But in the ſecond Caſe, x is always greater than it's Co-efficient; becauſe $x^2 - mx$ is equal to an affirmative Quantity. And in the third Caſe, x is always leſs than it's Co-efficient, becauſe mx and n are both negative. Hence, in compleating the Square, in the firſt and ſecond Caſes it matters not whether the Square of half the

Co-efficient, that is $\frac{m^2}{4}$, be greater or leſs than the abſolute Number n. But in the third Caſe it muſt always be greater than n, elſe there will be required to extract the ſquare Root of a negative Quantity, which is impoſſible.

SECONDLY, In the third Caſe, after the Square is compleated, if $\frac{m^2}{4}$ and n happen to be $=$, then, becauſe n is negative, $x^2 - mx + \frac{m^2}{4} = o$, whoſe Root being extracted $x - \frac{m}{2} = o$, and therefore $x = \frac{m}{2}$, conſequently $2x = m$:

THIRDLY, In the Solution of Equations of the third Form, the unknown Quantity will ſometimes have two affirmative Values, the one juſt, and the other more than juſt; for which Reaſon, this Sort of Equations are generally called ambiguous; but properly ſpeaking they are only particularly ſo. For when the Co-efficient of x, that is m, is leſs than $2x$, then there will be no Ambiguity at all: So this Ambiguity can only happen when m is greater than $2x$; for if m be equal to $2x$, then, by the laſt Obſervation, the Quantities $\frac{m^2}{4}$ and $-n$ will be equal, and conſequently deſtroy each other. To prove which, in this Equation $x = \frac{m}{2} \pm \sqrt{\frac{m^2}{4} - n}$. which is of the third Form, let us aſſume $2x$ for the Co-efficient of x inſtead of m, and ſuppoſing $2x - r$ any Co-efficient of x leſs than $2x$, but greater than x; and by ſubſtituting this Co-efficient, *viz.* $2x - r$, for m in this Equation $x = \frac{m}{2} \pm \sqrt{\frac{m^2}{4} - n}$. we ſhall have $x = \frac{2x - r}{2} \pm \sqrt{\frac{4x^2 - 4xr + r^2}{4} - n}$.

And for the Value of $-n$, form the original Equation with the unknown Quantity x and its Co-efficient $2x - r$; or, which comes all to the ſame Thing, ſubſtitute $2x - r$ for m in this Equation $x^2 - mx = -n$, and we ſhall have $-n = -x^2 + rx$; which being ſubſtituted in the aboveſaid Equation for $-n$. then

$$x = \frac{2x - r}{2} + \sqrt{\frac{4x^2 - 4xr - 4x^2 + 4rx + r^2}{4}}$$

which being contracted will become $x = \frac{2x - r}{2} \pm \sqrt{\frac{r}{4}}$. whence it is plain $x = \frac{2x - r}{2} + \frac{r}{2}$, and not $\frac{2x-r}{2}-\frac{r}{2}=x-r$. Therefore this Caſe is not ambiguous when the Value of the Co-efficient of the unknown Quantity is ſome Number greater than the ſaid Quantity, but leſs than its Double; which was to be proved.

BUT to prove that any Co-efficient of x greater than $2x$ will render the Caſe ambiguous; put $2x+r$ for any Co-efficient of x greater than $2x$, which being ſubſtituted for m in this Equation $x=\frac{m}{2}\pm\sqrt{\frac{m^2}{4}-n}$. as above ſpecified, it will become $x=\frac{2x+r}{2}\pm\sqrt{\frac{4x^2+4xr-4x-4xr+r^2}{4}}$. whence $x=\frac{2x+r}{2}\pm\sqrt{\frac{r^2}{4}}$. or $x=\frac{2x+r}{2}\pm\frac{r}{2}$; but $\frac{2x+r}{2}+\frac{r}{2}=x+r$, which is one affirmative Value of the unknown Quantity, but too much by r, the Exceſs of the Co-efficient above twice the Quantity ſought; and $\frac{2x+r}{2}-\frac{r}{2}=x$ is the other, the true Value of the Quantity required. Hence proceeds the Ambiguity which was to be proved.

SCHO.

SCHOLIUM.

From what is proved above, it appears, that $\frac{1}{2}$, the Difference between the Co-efficient of x and $2x$, that is $\frac{r}{2}$, is always equal to $\sqrt{\frac{m^2}{4}-n}$. and that ſeeing from the firſt Demonſtration $\frac{2x-r}{2}-\frac{r}{2}$ gives the unknown Quantity too little, and from the ſecond, $\frac{2x+r}{2}+\frac{r}{2}$ too much by r. Therefore when this Caſe is not ambiguous, always add $\sqrt{\frac{m^2}{4}-n}$. to $\frac{m}{2}$, and the Sum will give x the Quantity required; but when ambiguous, then always ſubtract $\sqrt{\frac{m^2}{4}-n}$. from $\frac{m}{2}$, and the Difference will give the Quantity ſought. Thus, in the former $x=\frac{m}{2}+\sqrt{\frac{m}{4}-n}$. and in the latter $x=\frac{m}{2}-\sqrt{\frac{m^2}{4}-n}$. But the Learner muſt know that the Ambiguity of this Caſe cannot be determined when the Equation is expreſſed literally; and for this Reaſon the Quantities $\frac{m}{2}$ and $\sqrt{\frac{m^2}{4} - n}$. are always connected together with a doubtful Sign. Thus, $x = \frac{m}{2} \pm \sqrt{\frac{m^2}{4} - n}$. See the third Caſe aforegoing.

Fourthly, In every quadratic adfected Equation, or indeed all compound Equations whatſoever, are as many Roots or Values of the unknown Quantity, either affirmative or negative, as the Index of the higheſt Power of the Quantity ſought contains *Units*; and when one of thoſe Roots is diſcovered, every Root beſides is found by dividing the whole Equation by that Root continually, or in quadratic adfected Equations, having found one Root, ſubtract it from the Co-efficient of their reſpective Equations, ^ and the Remainder will give the other Root. with a contrary sign

Fifthly and Laſtly, In the firſt and ſecond Caſes of quadratic Equations, one Root will be affirmative and the other a negative Quantity; but in the third Caſe, both Roots will be affirmative, tho' but one of them will anſwer the Queſtion, which muſt be taken according to the Conditions thereof; and in the other two Caſes, or indeed all

Equations whatſoever, the affirmative Quantity always ſolves the Problem, for the negative Quantities are of no other Uſe than to help to form the original Equation by

Thus much I preſume is ſufficient for what may be required in quadratic adfected Equations; from whence it appears, that this Equation $x^2 - ax = -c$ at the fourth Step in the Calculus to the foregoing Example, is of the third Form, and therefore from what hath been ſhewn above relating to this Sort of Equations, it is evident $x = \frac{a}{2} + \sqrt{\frac{a^2}{4} - c}$.

From the 47th of *Euclid*'s firſt Book it will eaſily appear that the Legs of a right-angled Triangle are equally related to the other Side, or Hypothenuſe. So that if inſtead of putting x for A B (*See Fig.* 8.) I ſuppoſe it = AC; then (*per Ax.* 1.) $a - x$ = AB, which Expreſſion is the very ſame as before, tho' x repreſents a different Part of a; and being ſtated according to the aboveſaid Propoſition of *Euclid*, will ſtand

$$\begin{array}{r|r|l}
\text{Thus,} & 1 & x^2+a^2-2ax+x^2=b^2.\\
1\;\; -a^2 & 2 & 2x^2-2ax=b^2-a^2\\
2\;\; \div\overline{2} & 3 & x^2-ax=\dfrac{b^2-a^2}{2}=-c.\\
3\;\; \mathrm{C}\square & 4 & x^2-ax+\dfrac{a^2}{4}=\dfrac{a^2}{4}-c.\\
4\;\; \omega\overline{2} & 5 & x-\dfrac{a}{2}=\sqrt{\dfrac{a^2}{4}-c}.\\
5\;\; +\dfrac{a}{2} & 6 & x=\dfrac{a}{2}\pm\sqrt{\dfrac{a^2}{4}-c}.
\end{array}\left.\vphantom{\begin{array}{l}x\\x\\ \dfrac{x}{x}\\ \dfrac{x}{x}\\ \dfrac{x}{x}\\ \dfrac{x}{x}\end{array}}\right\}\;\text{Seeing } a^2 \text{ is greater than } b^2\text{, it is evident their Difference will be negative, or } -c.$$

Now by comparing this Operation with that aforegoing, we ſhall find, that both the Form of the Calculus and Concluſion is the ſame in each, notwithſtanding different Quantities are the Scope of our Enquiry. And hence it is that ſome Geometrical Problems are ambiguous; and when ſuch occur, the great and incomparable Mathematician, Sir *Iſaac Newton*, adviſes, that when there happens to be ſuch an Affinity or Similitude of the Relations of two Terms to the other Terms of the Queſtion, that you ſhould be obliged, in making Uſe of either of them, to bring out Equations exactly alike; or that both, if they are made Uſe of together, ſhould bring out the ſame Dimenſions, and the ſame Form (only except-ing perhaps the Signs + and —) in the final Equation (which will be eaſily ſeen) then it will be the beſt Way to make uſe of neither of them, but in their Room to chuſe ſome third which ſhall bear a like Relation to both, as ſuppoſe the half Sum, or half Difference, or perhaps a mean Proportional, or any other Quantity relating to both indifferently and without alike. See the *Univerſal Arithmetic* Page 126.

THAT is, in this Example ſeeing the Similitude of the Relations of the two Terms or Legs AB and AC to the other Term or Hypothenuſe BC, is ſuch, that making uſe of either of them, brings out Equations exactly alike; therefore, for a more commodious Election of the Terms for the Calculus, by the foregoing Advice, chuſe ſome third Term, which, becauſe the Sum of them Legs are given, let be their Difference, for which put x, and by the proceeding *Lemma*, the greater Leg will be $\frac{a}{2} + \frac{x}{2}$, and the leſſer Leg $\frac{a}{2} - \frac{x}{2}$; both which being ſquared will

$$\text{become}\left\{\begin{array}{l|l} 1 & \frac{a^2}{4}+\frac{ax}{2}+\frac{x^2}{4}. \\ 2 & \frac{a^2}{4}-\frac{ax}{2}+\frac{x^2}{4} \end{array}\right\}\text{whoſe Sum}$$

$$\begin{array}{ll|l} & \text{is} & 3 \quad \frac{a^2}{2}+\frac{x^2}{2}=b^2. \\ 3 & \times\ 2 & 4 \quad a^2+x^2=2b^2 \\ 4 & -\ a^2 & 5 \quad x^2=2b^2-a\text{, a Simple Quadratic.} \\ 5 & \omega\ 2 & 6 \quad x=\sqrt{2b-a^2}. = AB - AC, \end{array}$$

or AC — AB, according as AB or AC repreſents the greater or leſs Leg.

Hence, by computing after this Method, it appears, that not only the final Equation is reduced to a ſimple Quadratic, but the Ambiguity of the Propoſition is likewiſe avoided; and therefore in ſome Queſtions it is more commodious to ſeek not the Quantities that are required, but ſome others from whence they may be found; as the Reader will find if he obſerves ſome of the foregoing and following Operations.

But to render the Ambiguity or Nature of this Example more conſpicuous, take the following Geometrical

CONSTRUCTION.

DRAW the Line BD (*See Fig.* 10.) equal to the Sum of the Legs AB and AC, in which take the Point e any where at Pleaſure; from which Point erect the Perpendicular eb equal to the Segment eD; from D, by the End of the Perpendicular eb, draw the Line Dbf of any convenient Length. This done, with the Hypothenuſe BC as Radius, and from B the Center deſcribe the Arch $g\,c\,C$, and from the Point C where the ſaid Arch cuts the Line Dbf, let fall the Perpendicular CA; and the Triangle required to be conſtructed is ABC.

DEMONSTRATION.

THE Lines be and CA being both drawn from the ſame Line Dbf perpendicular to the Line BD, it is evident the Triangles beD and CBD are ſimilar, and therefore $eD : eb :: AD : AC$. but eD and eb are equal by Conſtruction, conſequently AD is equal to AC. Whence it is plain AB + AC is = AB + AD = AD.

Q. E. D.

COROLLARY.

From the foregoing Conſtruction, ſeeing the Arch Ccg cuts the Line Dbf in another Point c, it will be eaſy to demonſtrate Geometrically the Reaſon of the Ambiguity in this Example. Thus, let fall the Perpendicular ca, and with a right Line join the Points B and c. Then by ſimilar Triangles $be : eD :: ca : aD$; but be and eD are equal by Conſtruction, therefore ca is equal to aD, conſequently $Ba + ca = Ba + aD = BA + CA$; whence the Triangles ABC and aBc are equal in all Reſpects, for BA is $= ca$, and $Ba = CA$. Now ſince from each of the Points of Interſection C, c, the ſame Triangle is produced, it is therefore indifferent which of the ſaid Points the Triangle is conſtructed from, ſince they both bear a like Relation to the Line BD.

EXAMPLE II.

Or if inſtead of the Sum of the Legs, as in the laſt Example, there had been given their Difference $= a$, and the Hypothenuſe $= b$ as before, to find the Legs: Then by the common Method of Computation put $x =$ the greater Leg, and (*per A*. 2.) the leſſer Leg will be $= x - a$. And by the 47th Propoſition of *Euclid*'s firſt Book, we ſhall have $BC^2 = AB^2 + AC^2$ (*See Fig*. 9.) That is,

	1	$b^2 = x^2 - 2ax + a^2 + x^2.$
$1 - a^2$	2	$2x^2 - 2ax = b^2 - a^2$
$2 \div \overline{2}$	3	$x^2 - ax = \frac{b^2 - a^2}{2}$
3 C□	4	$x^2 - ax + \frac{a^2}{4} = \frac{2b^2 - a^2}{4}$
$4\ \omega\ \overline{2}$	5	$x - \frac{a}{2} = \sqrt{\frac{2b^2 - a^2}{4}}.$
$5 + \frac{a}{2}$	6	$x = \frac{a}{2} + \sqrt{\frac{2b^2 - a^2}{4}}.$

the greater Leg, which was to be found, whence the leſſer Leg will be known. But by putting $x =$ the leſſer Leg, then the greater Leg will be $x + a$, and by the abovesaid Propoſition of *Euclid*, we ſhall have this

Equa-

Equation	1	$x^2 + 2ax + a^2 + x^2 = b^2$.
1 — a^2	2	$2x^2 + 2ax = b^2 - a^2$
2 $\div \overline{2}$	3	$x^2 + ax = \dfrac{b^2 - a^2}{2}$
3 C□	4	$x^2 + ax + \dfrac{a^2}{4} = \dfrac{2b^2 - a^2}{4}$
4 ω $\overline{2}$	5	$x + \dfrac{a}{2} = \sqrt{\dfrac{2b^2 - a^2}{4}}$.
5 — $\dfrac{a}{2}$	6	$x = -\dfrac{a}{2} + \sqrt{\dfrac{2a^2 - a^2}{4}}$.

SEEING that in the foregoing Operations their final Equations are the very ſame in both, except that in the former $\frac{a}{2}$ is affirmative, which in the latter is a negative Quantity; the third Term therefore to be made uſe of for the better Election of the Terms for the Calculation, muſt be the Sum of the Legs, becauſe their Difference is given, for which put x; then (*per Lemma following*) the greater Leg will be $\frac{x}{2} + \frac{a}{2}$, and the leſſer $\frac{x}{2} - \frac{a}{2}$, and the Sum of their Squares

is

$\frac{x^2}{2} + \frac{a^2}{2}$, which is equal to b^2 by the Figure.

$$\begin{array}{ll|l|l}
\text{Conſequently} & & 1 & \frac{x^2}{2} + \frac{a^2}{2} = b^2. \\
1 & \times \ \overline{2} & 2 & x^2 + a^2 = 2b^2 \\
2 & - a^2 & 3 & x^2 = 2b^2 - a^2 \\
3 & \omega \ \overline{2} & 4 & x = \sqrt{2b^2 - a^2}. = AB + AC
\end{array}$$

AND thus, by Help of the half Sum and half Difference, I have brought out the reſulting Equation in a ſimple quadratic Form, and freed of Ambiguity: tho' I might have omitted this Expreſſion; for no ſimple Equation, of what Kind ſoever, can be ambiguous.

GEOMETRICALLY.

[illegible]RAW the Line BD equal the given Difference of AB and AC (*See Fig.* 11.) to which add the Line D*e* of any convenient Length; from *e* erect the Perpendicular *be* equal to D*e*, and draw the Line D*bf* of any Length at Pleaſure; then from B ſet off the Hypotheнуſe BC to the ſaid Line in C, and let fall the Perpendicular CA, ſo is ABC the Triangle which was required.

DEM[illegible]

DEMONSTRATION.

BECAUSE the Lines AC and *eb* are both perpendicular to the Line D*e*, &c. the Triangles DAC and D*eb* are ſimilar, whence D*e* : *eb* : : DA : AC. but D*e* is = *eb* by Conſtruction, therefore DA is = AC. conſequently AB — DA = BD.

Q. E. D.

FARTHER, to ſhew the Utility of this Method of Sir *Iſaac Newton*'s, take another Example; and the better to conceive the Reaſon of expreſſing the half Sum and half Difference of any two Quantities, obſerve the following

LEMMA.

IF from half the Sum of any two Quantities be taken half their Difference, the Remainder will give the leſſer Quantity; b[illegible] added to the half Sum, gives the greater Quantity.

DEMONSTRATION.

PUT $x =$ the greater Quantity, and $y =$ the leſs; [illegible] make their Sum $x + y = s$, and Difference $x - y = d$. The half Sum will be $\frac{x+y}{2}=\frac{s}{2}$, and half Difference $\frac{x-y}{2}=\frac{d}{2}$. Now $\frac{x+y}{2}-\frac{x-y}{2}=\frac{s}{2}-\frac{d}{2}=y$. and $\frac{x+y}{2}+\frac{x-y}{2}=\frac{s}{2}+\frac{d}{2}=x$.

Q. E. D.

EXAMPLE III.

In the right-angled Triangle ABC (*See Fig.* 12.) there is given the Hypothenuſe AC $=a$, and BD, the Perpendicular let fall from the right Angle to the Hypothenuſe AC, $=b$. To find the Legs AB and BC; and the Segments of the Hypothenuſe AD and DC?

To ſolve this Example by the common Methods, put $x=$ one of the Segments, as ſuppoſe the greater, and (*per Ax.* 1.) the leſſer will be $=a-x$. Now, by the eighth Propoſition of *Euclid*'s ſixth Book, the Triangles ADB and BDC are ſimilar.

Whence

Whence That is,	1 2	AD : BD :: BD : DC. $x : b :: b : a - x.$
2 *Ergo*,	3	$ax - x^2 = b^2$ An Equation of the third Form.
3 ±	4	$x^2 - ax = -b^2$
4 C □	5	$x^2 - ax + \frac{a^2}{4} = \frac{a^2}{4} - b^2$
5 ɯ $\overline{2}$	6	$x - \frac{a}{2} = \sqrt{\frac{a^2}{4} - b^2}.$
6 + $\frac{a}{2}$	7	$x = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b^2}$, = A D or

DC, which ſoever is the longeſt. Having thus found the Segments of the Hypothenuſe, the Legs AB and BC will be found by Virtue of the 47th Propoſition of *Euclid*'s firſt Book; for in the Triangles ADB and BDC there is given the Legs AD, DB, and DC.

But if x be put for the leſſer *Seg*[illegible] then $a - x$ will be the greater; and ſeeing the Expreſſion is the ſame in both Suppoſitions, that is, $a - x$ repreſent indifferently the greater and leſs Segment, it is evident this Example is ambiguous, and therefore ſince the Sum of the Segments are given; let x repreſent their Difference, and *(per Lem-*

ma) the greater Segment will be $= \frac{a}{2} + \frac{x}{2}$, and the leſs $\frac{a}{2} - \frac{x}{2}$: And ſo by the above-cited Propoſition of *Euclid*, we ſhall have this

Equation, 1 | AD : BD : : BD : DC.
That is, 2 | $\frac{a}{2} + \frac{x}{2} : b :: b : \frac{a}{2} - \frac{x}{2}$
2 *Ergo*, 3 | $\frac{a^2}{4} - \frac{x^2}{4} = b^2$
3 × 4 | 4 $a^2 - x^2 = 4b^2$
4 + | 5 $x^2 = a^2 - 4b^2$
5 ω 2 | 6 $x = \sqrt{a^2 - 4b}$. the Difference

of the Segments of the Hypothenuſe AC, which being ſubſtituted for x in $\frac{a}{2} + \frac{x}{2}$ and $\frac{a}{2} - \frac{x}{2}$, the greater Segment will be equal to $\frac{a}{2} + \sqrt{\frac{a^2}{4} - b^2}$ and the leſſer $= \frac{a}{2} - \sqrt{\frac{a^2}{4} - b^2}$. which exactly agrees with the 7th Step of the laſt Operation. Hence it appears, that by ſubſtituting the Value of x found by this Method of Computation, for x in the Quantities expreſſed by the half Sum and half Difference, gives the very ſame

Theorems for finding thoſe Quantities, as tho' they were inveſtigated by the common Methods of *Algebra*.

GEOMETRICALLY.

On AC draw the Semicircle A*b*BC, draw alſo the Line *ac* parallel to AC at the Diſtance of the given Perpendicular BD, and where the ſaid Line *ac* cuts the Semicircle, as in the Point B or *b*, let fall the Perpendicular BD or *bd*; this done, draw the Lines AB and BC, or A*b* and *b*C, and ABC or A*b*C is the Triangle required. The Truth of this Conſtruction is evident from the Figure itſelf, and therefore needs no Demonſtration.

Having thus ſhewn what Quadratic adfected Equations are, and how they are brought to a Solution; with ſome uſeful Animadverſions on every Caſe, wherein, I preſume, is ſpecified all that is requiſite for finding that Root of an Equation that truly anſwers the Queſtion: I will now conclude this Synopſis, and ſo proceed to give the Solutions to the following Problems.

PROBLEM XXXIII.

Suppoſe two Towers, AB 180 *Feet high, and* CD 240, (*See Fig.* 13) *at the Diſtance* AC 360 *Feet. A Ladder is to be ſet upon the Line* AC, *at ſome Point, ſuppoſe in* E, *of ſuch a Length, as from thence it may reach the Top of both the Towers. We require the Point* E *in the Line of Diſtance, as alſo the Length of the Ladder* EB, *or* ED?

PUT $180 = a$, $240 = b$, $360 = c$, and $x =$ AE and (*per Ax.* 1.) $c - x =$ EC; but becauſe BE is equal to DE, by the 47th Propoſition of *Euclid*'s firſt Book, we ſhall have $AB^2 + AE^2 = EC^2 + DC^2$ which in

Species will ſtand thus,	1	$a^2 + x^2 = c^2 - 2cx + x^2 + b^2$.
1 $- x^2$	2	$a^2 = c - 2cx + b^2$
2 $+ 2cx$	3	$2cx + a = c^2 + b^2$
3 $- a^2$	4	$2cx = c^2 + b^2 - a^2$
4 $\div 2c$	5	$x = \frac{c^2 + b^2 - a^2}{2c} = 215 =$ AE,

which taken from 360 leaves $145 = $ EC, which gives the Point E required. Whence the Length of the Ladder will be found to contain 280.4 Feet.

W. W. D.

Conſtruction Geometrically.

JOIN the Points B and D with a right Line, which divide into two equal Parts in F, from which Point raiſe the Perpendicular FE, and where it cuts the Line AC, as in the Point E, there the Ladder muſt be ſet to reach to the Tops of both Towers. For BF is = FD, and FE common to both, therefore the Triangle BED is *Iſoſceles*, and conſequently BE is equal to ED.

Q. E. I. and *D.*

PROBLEM XXXIV.

In the Triangle ABC, *(See Fig.* 14.) *the ſeveral Sides* $AB = 13, AC = 14, BC = 15$ *are given; and the Perpendicular* BD *being drawn. Requir'd the Segments of the Baſis* AD, DC?

PUT $13 = a$, $14 = b$, $15 = c$, and $x =$ the Segment BD, then (*per Ax.* 1.) $c - x$ is equal the other Segment DC; and becauſe the Triangles ADB and ADC are right angled, and the Perpendicular AD common to both, we ſhall have (by Virtue of the 47th Propoſition of *Euclid*'s firſt Book, this

Equation	1	$AB^2 - BD^2 = AC - CD^2$. which in
Species is	2	$a^2 - x^2 = b - c^2 + 2cx - x^2$.
$2 \quad + x^2$	3	$a^2 = b - c^2 + 2cx$
$3 - b + c^2$	4	$2cx = a^2 - b^2 + c^2$
$4 \quad \div \quad 2c$	5	$x = \frac{a^2 - b^2 + c^2}{2c} = 6.6 =$ BD, and

$15 - 6.6 = 8.4 =$ the other Segment DC.

Q. E. I.

OR, by the Application of the 36th Propoſition of the third Book of *Euclid*'s *Elements*, the Segments D B and D C may be found by Proportion thus, as BC : AB + AC : : AC — AB : DC — DB, that is, as 15 : 13 + 14 (= 27.) : : 14 — 13 (= 1.) : $\frac{27}{15}$ (= DC — DB.) Now the Half of $\frac{27}{15}$ added to half 15 gives 8.4 = DC the greater Segment, and ſubtracting half $\frac{27}{15}$ from half 15, the Remainder gives 6.6 = DB the leſſer Segment, the ſame as before.

W. W. D.

SCHOLIUM.

FROM the foregoing Operation it may be obſerved from the third Step, that by tranſpoſing c^2 to the other Side the Equation, we ſhall have $a^2 + c^2 = b^2 + \underline{2cx}$. that is $AB^2 + BC^2 = AC^2 + 2 \times \overline{BC \times BD}$, an Equation that exactly agrees with the 13th Propoſition of the ſecond Book of *Euclid*; whence the Learner is to take Notice, that by carefully looking into the Calculations of Algebraical Problems, many curious and uſeful Theorems are often diſcover'd, and ſometimes new Properties of the Figure the Calculus belongs to, is brought to Light; and to this Property of Analytical Operations, both Arithmetic and Geometry chiefly owe their greateſt Improvements.

PROBLEM XXXV.

In the obtuſe angle Triangle DEF, (*See Fig,* 15.) *the ſeveral Sides are given,* viz. DE 11, EF 13, DF 20; *and the Perpendicular* FG, *being let fall upon the Baſis produced. Required the Prolongation of the Baſis* EG.

PUT $20 = a$, $13 = b$, $11 = c$, and x = EG the Prolongation of the Baſis DE; then (*per Ax.* 2.) $c + x$ = DG; now becauſe the Triangles DFG and EGF are right angled, and the Perpendicular FG common to both, it is evident (by the 47th Propoſition of *Euclid's* firſt Book) that $DF^2 - DG^2 = EF^2 - EG^2$. that is,

in Species	1	$a^2 - c^2 - 2cx - x^2 = b^2 - x^2$
1st $+ x^2$	2	$a^2 - c^2 - 2cx = b^2$
2 $\pm$	3	$2cx = a^2 - c^2 - b^2$
3d $\div 2c$	4	$x = \frac{-b^2 + a^2 - c^2}{2c} = 5 =$ EG.

Q. E. I.

SCHOLIUM.

At the ſecond Step by tranſpoſing $-c^2 - 2cx$ to the other Side of the Equation, it will be $a^2 = b^2 + c^2 + 2cx$, the ſame with the 12th Propoſition of *Euclid*'s ſecond Book of Geometry.

GEOMETRICALLY.

Upon the Side DF draw the Semicircle DGF, and produce out the Side DE 'till it cuts the ſaid Semicircle in the Point G; then the Segment EG will be the Prolongation required.

DEMONSTRATION.

Draw the Line FG, and (by the 31ſt Propoſition of *Euclid*'s third Book) the Angle DGF is a right Angle; conſequently the Line FG is perpendicular to the Side DE produc'd, to G. Therefore E G is the Prolongation ſought.

Q. E. D.

PROBLEM XXXVI.

In the Rectangle ABCD, (*See Fig.* 16.) *is given the Difference between the Length* AB *and the Diagonal* BD, *that is* D E = 2; *and likewiſe the Difference between the Breadth* A D *and the Diagonal* BD, *that is*, FE = 9. *Required the Sides of the Rectangle* AB, AD?

PUT 9 = a, 2 = b, and x = AD = DF; then DB = $a + x$, and AB = EB = $a + x - b$. Now the Triangle ABD being right-angled, there will be had (by the 47th Propoſition of *Euclid*'s firſt Book) this

Equation,	1	$DB^2 = AB^2 + AD^2$.
That is,	2	$a^2 + 2ax + x^2 = x^2 + a^2 + 2ax - 2ab + x^2 - 2bx + b^2$.
2 $\underline{+}$	3	$x^2 - 2bx = 2ab - b^2$
3 C$\underline{\square}$	4	$x^2 - 2bx + b^2 = 2ab$
4 ω 2	5	$x - b = \sqrt{2ab}$.
5 + b	6	$x = \sqrt{2ab}. + b = \sqrt{2 \times 9 \times 2}$.

+ 2 = 8 = AD, the Breadth of the Rectangle,

angle, which being known, the Length AB will be found $= 15$.

Q. E. I.

OR put $x =$ EF; then (by the Figure) $a + x =$ AB, $b + x =$ AD, and $a + b + x =$ BD, which being ſtated as above directed, it will

ſtand thus

$$\begin{array}{ll|l|l}
 & & 1 & a^2+2ab\begin{array}{l}+2a\\+2b\end{array}x+b^2+x^2\\
 & & & \quad = \begin{array}{l}a^2+2a\\b^2+2b\end{array}x\begin{array}{l}+x^2\\+x^2\end{array}\\
1 & \pm & 2 & x^2=2ab\\
\hline
2 & \omega\ \ 2 & 3 & x=\sqrt{2ab.}=\sqrt{2\times9\times2.}=6=\mathrm{EF},
\end{array}$$

whence the Side AB $= 15$, AD $= 8$, and BD $= 17$. *W. W. R.*

SINCE the End and Deſign of Algebraical Solutions is to bring out the moſt elegant and ſimple Equations poſſible; and becauſe ſome Ways are much neater than others; wherefore (ſays the great Sir *Iſaac Newton*, in his *Univerſal Arithmetic*, Page 98) if the Method you take from your firſt Thoughts for ſolving a Problem, be ill accommodated to Computation, you muſt again conſider the Relation of the Lines, until you have hit on a Way as fit and elegant as poſſible. For thoſe Ways that offer themſelves at firſt Sight, may often create ſufficient Trouble if they are made Uſe of. Thus, it would not have been more difficult to have fallen upon the following Method than upon one of the precedent ones. For having put $x = DB$, then $AB = x - b$, and $AD = x - a$; whence, by the Property of the Figure or Triangle ABD, this Equation will be

obtained	1	$x^2 = x^2 - 2bx + b^2 + x^2 - 2ax + a^2.$
1 $+$	2	$x^2 - 2bx - 2ax = - a^2 - b^2$
2 $\overline{C\Box} -$	3	$x^2 - 2bx - 2ax + b^2 + 2ba + a^2 = 2ab$
3 ω 2	4	$x - a - b = \sqrt{2ab}.$
4 $+ a + b$	5	$x = a + b + \sqrt{2ab.} = 17 = DB$, &c.

Hence, by computing after this Rate, I have fallen into larger and more perplexed Algebraical Terms, than either of the former Methods.

But of theſe three Ways of Computation, the ſecond Operation is much more ſimple than either of the other two; for in thoſe Calculations their reſulting Equations come out in an adfected quadratic Form, whereas this produces a ſimple Quadratic only. From which may be drawn the following

SCHOLIUM.

Seeing, at the ſecond Step, there comes out this Equation $x^2 = 2ab$, which turned into an Analogy, will be $2a : x :: x : b$, or $2b : x :: x : a$; that is, 2FB : EF :: EF : DE, or 2DE : EF :: EF : FB, whence x or EF is a mean Proportional between $2a$ and b, or $2b$ and a; or between 2FB and DE, or 2DE and FB. Hence it will not be difficult to conſtruct this Problem Geometrically, which I ſhall leave to exerciſe the ingenious Analyſt.

PROBLEM XXXVII.

In a Rectangle DEFG, (*See Fig.* 17.) *the right Line* DK *is drawn from the Angle* D *to the oppoſite Side, cutting the Diagonal* EG *at right Angles in* H: *And there is given the Segment* HK $=2$, *and* HE $= 16$. *Required the Sides of the Rectangle?*

To ſolve this Problem, put $16 = a$, $2 = b$, and $x = \text{GH}$; and becauſe the Triangles KGD and GDE are right angled, and DK and GE cut each other at right Angles in H, by the Queſtion. Therefore by ſimilar

Triangle		1	$KH : GH :: GH : HD.$
		2	$GH : HD :: HD . HE.$
That is,		3	$b : x :: x : \frac{x^2}{b}.$
		4	$x : \frac{x^2}{b} :: \frac{x^2}{b} : a.$
4	Ergo,	5	$ax = \frac{x^4}{b^2}$
5	$\times b$	6	$ab^2 x = x^4$
6	$\div x$	7	$x^3 = ab^2$
7	ω 3	8	$x = \sqrt[3]{ab^2}. = \sqrt{16\times 2\times 2.} = 4.$

Having thus found GH = 4, DH will be given = 8; whence (by the 47th Propoſition of *Euclid*'s firſt Book) the Length PE will be = 18.02; and the Breadth DG = 8.75, the Sides of the Rectangle.

W. W. F.

OBSERVATION.

THIS Problem, if rightly conſider'd, is no other than the finding two mean Proportionals between two given Quantities; for ſeeing HK : HG :: HG : HD :: HD : HE; it is evident the Lines HG and HD, are two mean Proportionals between the given Lines HK and HE. Put x = the firſt Mean, and, from the Laws of Geometrical Progreſſion, the ſecond will be $= \frac{x^2}{a}$; having thus obtained the two Means in Species, multiply them together, and their Product will be $\frac{x^3}{a}$, which is equal to the Rectangle of a into b; that is, $\frac{x^3}{a} = ab$. This Equation multiplied by a, becomes $x^3 = a^2b$, and extracting the Cube Root $x = \sqrt[3]{a^2b}$. the ſame as before. But if it be required to find three mean Proportionals between a and b; and putting $x =$ firſt Mean, the ſecond will be $\frac{x^2}{a}$, and the third $= \frac{x^3}{a}$; multiply x by $\frac{x^3}{a^2}$, or $\frac{x^2}{a}$ into itſelf, and there will be had $\frac{x^4}{a^2}$, which is $= a \times b$; that is, $\frac{x^4}{a} = ab$, which multiplied by a^2, $x^4 = a^3b$, and extracting the fourth Root, or the ſquare Root of the ſquare Root, we ſhall have $x = \sqrt[4]{a^3b}$, or $x = \sqrt{\sqrt{a^3b}}$.. Or if it were required to find four mean Proportionals between a and b, then if x repreſents the firſt

Mean, the ſecond will be $\frac{x^2}{a}$, the third $\frac{x^3}{a^2}$, and the fourth $\frac{x^4}{a^3}$; and the Form of the Equation is $\frac{x^5}{a^3} = ab$; which reduced, and the Root extracted, gives $x = \sqrt[5]{a^4b}$. Now from theſe Equations $x^3 = a^2b$, $x^4 = a^3b$, and $x^5 = a^4b$, it appears, that x, the firſt mean Proportional, will always be raiſed to a Power, whoſe Index is the Number of Means ſought *more* 1, and a, the firſt Term of the Series, will be raiſed to that Power whoſe Index is the Number of Means required. Hence, putting $n =$ any Number of mean Proportionals between a and b, we ſhall have this univerſal Theorem for ſolving all Queſtions of this Nature, *viz.* $\sqrt[n+1]{a^nb}$. but to effect the ſame Logarithmetically, put $l:a =$ Logrithm of a, and $l:b =$ Logarithm of b, and we ſhall have this Logarithmetical Theorem, *viz.* $\frac{n \times l:a + l:b}{n+1}$, either of which Theorems gives the firſt mean Proportional of any Geometrical Series whatſoever. But the latter, for its Facility and Expedition in Calculation, is far preferable to the former.

Because moſt of our Writers of Arithmetic have only ſhewn how to extract the ſquare and cube Roots of Numbers, it may not be improper, in this Place, to give the Reader a Rule by which all Roots may be extracted univerſally, taken from Sir *Iſaac Newton*'s *Arithmetic*, Page 31.

RULE.

EVERY third Figure beginning from Unity is firſt of all to be pointed, if the Root to be extracted be a Cubic one; *or every fifth, if it be a* Quadrato Cubic, *or of the fifth Power,* &c. *and then ſuch a Figure is to be writ in the Quotient, whoſe greateſt Power* (i. e. *whoſe* Cube, *if it be a* Cubic Power, *or whoſe* Quadrato-Cube, *if it be the* fifth Power, *&c.*) *ſhall either be equal to the Figure or Figures before the firſt Point, or the next leſs; and then having ſubtracted that Power, the next Figure will be found by dividing the Remainder augmented by the next Figure of the Reſolvend, by the next greateſt Power of the Quotient, multiplied by the Index of the Power to be extracted, that is, by the* triple Square *of the Quotient if the Root be a* Cubic one; *or by the* Quintiple Biquadrate, *i. e.* five Times the Biquadrate *if the Root be of the* fifth Power, *&c. And having again ſubtracted the greateſt Power* *of the whole Quotient from the firſt Reſolvend, the third Figure will be found by dividing that Remainder augmented by the next Figure of the Reſolvend, by the next greateſt Power of the whole Quotient multiplied by the Index of the Power to be extracted*; and ſo on *in infinitum.*

NOTE. When the Biquadratic Root is to be extracted, you may extract twice the ſquare Root, for the ſquare Root of x^4 is x^2, and the ſquare Root of x^2 is x. And when the Root of the ſixth Power is to be extracted, you may firſt extract the ſquare Root, and then the *Cube Root* out of that ſquare Root, for the ſquare Root of x^6 is x^3, and the Cube Root of x^3 is x. *And the ſame is to be obſerved in other Roots, whoſe Indexes are not prime Numbers.*

PROBLEM XXXVIII.

Let there be a Circle, whoſe Diameter is AB, (*See Fig.* 18.) *with another leſs Circle whoſe Diameter* AC, *touches within in* A: *and from the Center of the greater Circle* D, *draw the Radius* DE *at right Angles to* AB, *cutting the Periphery of the leſſer Circle in* F. *Now there is given*

BC

BC (*the Difference of the Diameters*) $= 9$, *with the Segment* EF $= 5$. *Required the Diameters* AB, AC *of the ſaid Circles?*

Because the Circles AEB and AFC touch each other within, the Center of the leſſer Circle (by the 11th Propoſition of *Euclid*'s third Book) will be in the Diameter of the great Circle drawn from the Point of Contact. Wherefore putting $x = DB = DE = DA$ the Radius of the greater Circle, $9 = a$, and $5 = b$; then (*per Figure*) $DE = x - b$, and $DC = x - a$; now (by the 13th Propoſition of *Euclid*'s ſixth Book) it

will be	1	$AD \times DC = DF^2$. which expreſs'd
in Species	2	$x^2 - ax = x^2 - 2bx + b^2$.
2 $- x^2$	3	$-ax = -2bx + b^2$
3 $+ 2bx$	4	$2bx - ax = b^2$
4 $\div 2b - a$	5	$x = \dfrac{b}{2b - a} = \dfrac{5 \times 5}{2 \times 5 - 9} = 25$

$=$ DB. Conſequently AB the Diameter of the greater Circle is 50, which being known, the Diameter AC of the leſſer Circle will be $= 41$ $(= 50 - 9)$ *Q. E. I.*

Or thus,

From this evident Principle, that the Difference of the Halves of any two Quantities is equal to the half Difference of the Quantities themſelves; it follows, that PD, the Difference between the two Radii AD and AP, is equal $\frac{a}{2}$ the half Difference of the Diameters AB and AC. Whence putting $x =$ AP the Radius of the leſſer Circle, then AD = DE = DB $= x + \frac{a}{2}$, DC $= x - \frac{a}{2}$, and DF = DE — FE $= x + \frac{a}{2} - b$; which, by the aboveſaid Propoſition of *Euclid*, will be brought to an Equation in the very ſame Manner as Step the ſecond of the preceding Operation ſpecifies.

Thus, | 1 | $AD \times DC = DF^2$. which expreſſed in Species, will

be | 2 | $x^2 - \frac{a^2}{4} \left(= x + \frac{a}{2} \times x - \frac{a}{2}.\right) = x^2 + ax - 2bx - \frac{a^2}{4} - ab + b^2.$

2 $\pm$ | 3 | $2bx - ax = \frac{a^2}{2} - ab + b^2,$

3 $\div 2b - a$ | 4 | $x = \frac{\frac{a^2}{2} - ab + b^2}{2b - a} = \frac{b^2}{2b - a} - \frac{a}{2}$

$= 20.5 =$ AP, which being known, the Diameter AC will be twice 20.5, that is, = 41. and 41 + 9 = 50 the Diameter AB the ſame as before.

W. W. D.

PROBLEM XXXIX.

Two Companions have got a Parcel of Guineas; ſays A *to* B, *if you will give me one of your Guineas, I ſhall have as many as you will have left. Nay, replies* B, *if you will give me one of your Guineas, I ſhall have twice as many as you will have left. How many Guineas had each of them?*

PUT x = A's Number of Guineas, and the Number of Guineas B had will be = $x + 2$; and by the Condition of the Queſtion, we ſhall obtain this

Equation,	1	$x + 3 = 2x - 2$.
1 $- x$	2	$3 = x - 2$
2 $+ 2$	3	$x = 5$, hence A had 5 Guineas

and B had 7. *Q. E. I.*

PROBLEM

PROBLEM XL.

A certain Perſon bought two Horſes, with the Trappings, which coſt 100 *Pounds; which Trappings if laid on the firſt Horſe* A, *both Horſes will be of equal Value: but if the Trappings be laid on the other Horſe, he will be double the Value of the firſt. How much did the ſaid Horſes coſt?*

To ſolve this Problem put 100 = a, x the Value of the ſecond Horſe and Trappings together, and (by *Ax.* 1.) the firſt Horſe will be $= a - x$; but by the Queſtion, double the Value of the firſt Horſe is equal the Price of the ſecond Horſe and Trappings together; whence this Query will be reduced to the following Equation, which being ſtated will ſtand

Thus,		
	1	$x = 2a - 2x.$
1 + $2x$	2	$3x = 2a$
2 ÷ 3	3	$x = \frac{2a}{3} = \frac{2 \times 100}{3}$ $66\frac{2}{3}$ Pounds,

and $100 - 66\frac{2}{3} = 33\frac{1}{3}$ Pounds the Price of the firſt Horſe; from $66\frac{2}{3}$ take $33\frac{1}{3}$ and the Remainder is $33\frac{1}{3}$, which divided by 2 the

Quotient is $16\frac{2}{3}$ Pounds the Price of the Trappings, and the Sum of $33\frac{1}{3}$ and $16\frac{2}{3}$ is 50 Pounds what the ſecond Horſe coſt.

NUMERICALLY.

To ſolve this Problem by Numbers, reaſon thus; ſince the Trappings laid on the firſt Horſe makes both the Horſes of equal Value, and becauſe both the Horſes and Trappings together coſt 100 Pounds, the ſecond Horſe conſequently muſt coſt 50; but (by the Queſtion) if the Trappings be laid on the other Horſe, he will be double the Value of the firſt, therefore the firſt Horſe coſt $\frac{1}{3}$ of 100 which is $= 33\frac{1}{3}$ Pounds; now take $33\frac{1}{3}$ from 50, or $50 + 33\frac{1}{3}$ from 100, the Remainder will be $16\frac{2}{3}$ Pounds, the Price of the Trappings. *W. W. D.*

PROBLEM XLI.

A Vintner has two Sorts of Wine, viz. A *and* B: *which if mixed in equal Parts, a Flaggon of mixed will coſt* 15 *Pence; but if they be mixed in* ſeſqui-alter *Proportion, as if you ſhould take two Flaggons of* A *as often as you take three of* B, *a Flaggon will coſt* 14 *Pence. Required the Price of each Wine ſingly?*

Seeing by the former Part of the Queſtion, a Flaggon of Wine of equal Mixture coſt 15 Pence, two Flaggons of the ſame Mixture will coſt 30 Pence; and becauſe, by the latter Part of the Queſtion, two Flaggons of the Wine A mix'd with three Flaggons of the Wine B coſt fourteen Pence a Flaggon, it is evident that five Flaggons of this Wine comes to 70 Pence. Put $30 = a$, $70 = b$, and $x =$ the Price of a Flaggon of Wine A, and a Flaggon of the Wine B will coſt $a - x$ Pence; and ſince the Sum of the Prices of two Flaggons of A or $2x$, and three Flaggons of B or $3a - 3x$, added together, is equal to 70 Pence, or b, we ſhall

$$\begin{array}{l|l|l}
\text{have} & 1 & 2x + 3a - 3x = b. \\
1 + 3x & 2 & 2x + 3a = b + 3x \\
2 - b & 3 & 3x = 3a - b + \underline{\ 2x\ } \\
3 - 2x & 4 & x = 3a - b = 3 \times 30 - 70 = 20
\end{array}$$

Pence, which taken from 30 leaves 10 Pence, the Price of a Flaggon of the Wine B.

W. W. R.

I have hitherto given the Arithmetical Solution to many of the foregoing Problems, and ſhall obſerve the ſame Order in ſome of the following Propoſitions alſo; purely to ſhew the young Analyſt, that ſome Queſtions that are propoſed in Algebra, may, by mature Conſideration, be ſolved by Arithmetical Principles only.

HENCE, in this Queſtion, it will be no hard Taſk to find the Price of a Flaggon of each Sort of Wine independant of Algebra, and that too, even by the moſt ſimple Laws of Arithmetic. Thus, becauſe the Wine when mix'd with equal Quantities of each Sort coſt 15 Pence a Flaggon, it is evident that two Flaggons of A mixed with two Flaggons of B will coſt 60 Pence; and when mix'd in *ſeſqui-alter* Proportion, it is plain that five Flaggons of ſuch a Mixture, at the Rate of 14 Pence the Flaggon, will amount to 70 Pence: Now, ſubtracting 60 from 70, that is, ſubtracting the Price of two Flaggons of A mix'd with two Flaggons of B, from the Price of two Flaggons of A mix'd with three Flaggons of B, the Remainder is ten Pence the Price of a Flaggon of the Wine B, whence a Flaggon of the other Sort of Wine A will coſt twenty Pence. The Prices required. Thus you ſee, that by a right Way of thinking, ſome Things that may ſeem difficult to Algebraic Computation, may be ſolved by the eaſieſt Rules in Arithmetic.

PROBLEM XLII.

A Son aſk'd his Father how old he was; his Father anſwer'd him thus; If you take away 5 from my Years, and divide the Remainder by 8, the Quotient will be $\frac{1}{3}$ *of your Age; but if you add 2 to your Age, and multiply the Whole by 3, and then ſubtract 7 from the Product, you will have the Number of Years of my Age. What was the Age of the Father and the Son?*

PUT x = the Father's Age, and (by the Queſtion) the Son's Age will be $\frac{x-5}{8} \times 3$ or $\frac{3x-15}{8}$; but by adding 2 to the Son's Age, and multiplying that Sum by 3, and 7 ſubtracted from the Product; this laſt Remainder is equal the Father's Age, or x.

$$\begin{array}{lll|l|l} \text{Therefore,} & & & 1 & \frac{9x-53}{8} = x \\ 1 & \times & 8 & 2 & 9x - 53 = 8x \\ 2 & + & 53 & 3 & 9x = 8x + 53 \\ 3 & - & 8x & 4 & x = 53 \text{ Years, the Father's Age.} \end{array}$$

Hence the Son's Age will be found to be 18, the Anſwer which was required.

PROBLEM XLIII.

To find out two Numbers, to the Sum whereof if you add 6, the Whole ſhall be double the greater; and if you ſubtract 2 from their Difference, the Remainder will be half of the leaſt.

PUT $x=$ the greater Number, and y the leſſer; which being ſtated according to the Conditions of the Queſtion, will ſtand

Thus,	1	$x+y+6=2x.$
	2	$x-y-2=\frac{y}{2}.$
$2\times\overline{2}$	3	$2x-2y-4=y$
$1-x-\overline{6}$	4	$y=x-6.$ Subſtitute $x-6$ for y in the third Step,
and	5	$8=x-6$
$5+\overline{6}$	6	$x=14$ the greater Number,

and by the fourth Step, the leſſer will be found $=8$. Which was to be ſought.

NOTE, This Method of exterminating Quantities by ſubſtituting their Adequates for the Quantities themſelves in ſome other Equation, is called ſubſtitutive Algebra. And if the Learner deſires to ſee more of this Kind of Calculation, let him read Sir *Iſaac Newton*'s *Univerſal Arithmetic* from Page 60 to 66; where he will meet with the beſt Inſtructions of this Kind.

PROBLEM XLIV.

To find two Numbers, the Product whereof is 240, *and the Triple of the greater divided by the leſs is* 5.

Put $240 = a$, $5 = b$, $x =$ the greater Number, and (*per Ax.* 3.) the leſſer Number will be $= \frac{a}{x}$. The Triple of the greater Number is $3x$, which divided by the leſſer the Quotient is $\frac{3x^2}{a}$, which, by the Queſtion, is equal to b.

Whence, | 1 | $\frac{3x^2}{a} = b.$

1 ÷ $\frac{3}{a}$ | 2 | $x^2 = \frac{ab}{3}.$

2 ww $\overline{2}$ | 3 | $x = \sqrt{\frac{ab}{3}} = \sqrt{\frac{240 \times 5.}{3}} = 20$

the greater Number, and the leſſer will be $\frac{240}{20} = 12$. *Q. E. I.*

PROBLEM XLV.

Two Men have a Mind to purchaſe a Houſe rated at 1200 *Pounds; ſays* A *to* B, *if you will give me* $\frac{2}{3}$ *of your Money, I can purchaſe the Houſe alone; but ſays* B *to* A, *if you will give me* $\frac{1}{4}$ *of yours, I ſhall be able to purchaſe the Houſe. How much Money had each of them?*

PUT $1200 = a$, $x =$ A's Money, and $\frac{2}{3}$ of B's Money will be $a - x$, to which add $\frac{1}{2}$ itſelf, and the Sum is $\frac{3a - 3x}{2}$ the Money that B had; now, by the Queſtion, $\frac{1}{4}$ of A's Money added to B's Money, that Sum will purchaſe the Houſe, whence it is plain

that	1	$\frac{3x}{4} + \frac{3a - 3x}{2} = a.$
1 $\times$ 4	2	$6a - 3x = 4a$
2 +	3	$3x = 2a$
3 $\div$ 3	4	$x = \frac{2a}{3} = \frac{1200 \times 2}{3} = 800$ Pounds

A's Money, hence B had 600. for $\frac{2}{3}$ of 600 added to 800, or $\frac{3}{4}$ of 800 added to 600, either Sum will be 1200.

W. W. F.

PROBLEM XLVI.

Some young Men and Maids had a Reckoning of 37 Crowns to pay for a Treat, and this was their Conditions, that every young Man ſhould pay three Crowns, and every Maid two. Now, if there had been as many young Men as there were Maids, obſerving the ſame Conditions, the Reckoning would have come to four Crowns leſs than it did. How many young Men and Maids were there?

Put $37 = a$, $x =$ the Number of young Men, and the Number of Crowns they paid is $3x$, whence, according to the firſt Condition of the Queſtion, the Maids paid $a - 3x$ Crowns, and becauſe they paid two Crowns a piece, the Number of Maids will be expreſſed by this Fraction $\frac{a - 3x}{2}$. Now, by the ſecond Condition, there being ſuppos'd $\frac{a - 3x}{2}$ young Men, and x Maids, and the Men pay-ing three Crowns each, and the Maids two, the Number of Crowns the Men paid will be $\frac{3a-9x}{2}$, and the Maid $2x$ Crowns, the Sum of which, *viz.* $\frac{3a-9x}{2} + 2x$, by the Queſtion, is equal to $a - 4$.

Therefore,	1	$\frac{3a-9x}{2} + 2x = a - 4.$
$1 \times \overline{2}$	2	$3a - 5x = 2a - 8$
$2 \; \overline{+}$	3	$5x = a + 8$
$3 \div \bar{5}$	4	$x = \frac{a+8}{5} = \frac{37+8}{5} = 9$ the

Number of young Men, hence the the Number of Maids will be found to be 5. for $\overline{9 \times 3} + \overline{5 \times 2} = 37$, and $\overline{5 \times 3} + \overline{9 \times 2} = 33 = 37 - 4$. *Q. E. I.*

PROBLEM XLVII.

A General, who had fought a Battle, upon reviewing his Army, whoſe Foot was thrice the Number of his Horſe, finds that before the Battle $\frac{1}{12} - 120$ *of his Foot had deſerted, and of his Horſe,* $\frac{1}{20} + 120$, *beſides* $\frac{1}{4}$ *of his whole Army were ſent into Garriſons (reckoning the Sick*

Sick and Wounded) and $\frac{3}{8}$ of his Army remained; the reſt, who were wanting, being either ſlain or taken Priſoners; now, if you add 3000 *to the Number of the Slain, the Sum will be equal to half the Foot he had at the Beginning. What were the Numbers of each?*

For the Number of Horſe put x, then $3x$ is equal the Number of the Foot, and $4x$ will be the whole Army; the Number of the Foot deſerted $\frac{3x}{12} - 120$, and of the Horſe $\frac{x}{20} + 120$, and likewiſe $\frac{1}{4}$ Part of the whole Army ſent into Garriſon is x; now the Sum of the Horſe and Foot deſerted, together with $\frac{1}{4}$ Part of the whole Army is $\frac{3x}{12} + \frac{x}{20} + x$, or $\frac{13x}{10}$; to which add $\frac{3}{8}$ of the whole Army, *viz.* $\frac{12x}{8}$, and the Sum will be $\frac{13x}{10} + \frac{12x}{8}$, or $\frac{14x}{5}$. which Sum taken from $4x$ ($=$ the whole Army) the Remainder will be $4x - \frac{14x}{5}$ or $\frac{6x}{5}$ equal the Number of the Slain or taken

Priſoners. Whence, by the Queſtion, $\frac{6x}{5}$ + 3000 is equal to half the Foot he had at the Beginning.

Therefore, 1 | $\frac{6x}{5}+3000=\frac{3x}{2}$.
1 × 5 | 2 | $6x+15000=\frac{15x}{2}$
2 × 2 | 3 | $12x+30000=15x$
3 − 12x | 4 | $3x=30000$
4 ÷ 3 | 5 | $x=10000$ the Number of the

Horſe, and the Number of the Foot will be found to be 30000, whence there was 12000 ſlain or taken Priſoners, and the whole Army he had at firſt was 40000 Men.

W. W F.

PROBLEM XLVIII.

To divide 100 *twice into two Parts, ſo that the major Part of the firſt Diviſion may be treble the minor Part of the ſecond Diviſion; and the major Part of the ſecond may be double the minor Part of the firſt.*

PUT $100 = a$, $x =$ the major Part of the firſt Diviſion, and (*per Ax.* 1.) the minor Part will be $= a - x$; and, by the Queſtion, the major Part of the ſecond Diviſion will be $= 2a - 2x$, and minor Part $= \frac{x}{3}$. Now the Sum of the two Parts of the ſecond Diviſion (*per Query*) is equal to a, whence this Problem will be reduced to the following Equation,

Viz.		1	$2a - 2x + \frac{x}{3} = a.$
1	$\times \overline{3}$	2	$6a - 5x = 3a$
2	$+$	3	$5x = 3a$
3	$\div \overline{5}$	4	$x = \frac{3a}{6} = \frac{3 \times 100}{5} = 60$ the major

Part of the firſt Diviſion, which being known, the minor Part will be $= 100 - 60 = 40$; hence the major Part of the ſecond Diviſion will be found to be $100 - 60 \times 2 = 80$, and the minor Part $= 100 - 80$ (or $\frac{60}{3}$.) $= 20$, the Diviſions which were required.

PROBLEM XLIX.

To divide 30 *twice into two Parts, ſo that the major Part of the firſt Diviſion with the minor of the ſecond may be* 33; *and the Sum of the minor Parts ſubtracted from the Sum of the major, may leave* 14 *remaining.*

PUT $30 = a$, $14 = b$, $33 = c$, $x =$ the major Part of the firſt Diviſion, and (*per Ax.* 1.) the minor Part will be $= a - x$; but in the ſecond Diviſion the minor Part will be $= c - x$, and the major Part $= a - c + x$. Having thus expreſs'd the Parts of both Diviſions, the Sum of the major Parts is $2x + a - c$, from which take the Sum of the minor, *viz.* $a + c - 2x$, and (by the Queſtion) the Remainder is equal to b; whence there will be obtained the following Equation,

	Viz.	1	$4x - 2c = b.$
1	$+ 2c$	2	$4x = b + 2c$
2	$\div \overline{4}$	3	$x = \frac{b + 2c}{4} = \frac{14 + 2 \times 33}{4} = 20$

the major Part of the firſt Diviſion, which taken from 30 there remains 10 = the minor; and again take 20 from 33, the Remainder, *viz.* 13 gives the minor Part of the Second Diviſion, which Part taken from 30 leaves 17 = the major, which was to be divided.

PROBLEM L.

A Man, his Wife, and his Son's Ages make up 96 Years, ſo that the Husband's and Son's Years together make the Wife's + 15; *but the Wife's and the Son's make the Husband's* + 2. *What was the Age of each?*

For the Husband's Age put *x*, for the Wife's put *y*, and for the Son's put *z*.

Then	1	$x+y+z=96.$	by the Queſtion.
	2	$x+z=y+15.$	
	3	$y+z=x+2.$	
1 — 3	4	$x=96-x-2$	
4 + x	5	$2x=94$	
5 ÷ 2	6	$x=47$ Years, the Huſband's Age.	
1 — 2	7	$y=96-y-15$	
7 + y	8	$2y=81$	
8 ÷ 2	9	$y=40\frac{1}{2}$, the Wife's Age.	
6 + 9	10	$x+y=87\frac{1}{2}$	
1 — 10	11	$z=8\frac{1}{2}$ the Son's Age.	

Q. E. I.

37
23
14

The

The ſame otherwiſe.

THUS, put $x =$ the Husband's Age as before, and (*per Ax.* 1.) the Wife's and Son's Ages together will be $96 - x$; which, by the Queſtion, is equal to the Husband's Age $+ 2$. Whence we have this

Equation, | 1 | $96 - x = x + 2$.
1 $+$ | 2 | $2x = 94$
2 $\div$ 2 | 3 | $x = 47$ Years, the Husband's

Age. Hence $96 - 47$ is equal to the Wife's and Son's Ages together, that is, 49. Now let x repreſent the Wife's Age, and the Son's Year's will be $49 - x$; then, by the former Part of the Queſtion, this Problem will be reduced to the following

Equation, | 1 | $47 + 49 - x = x + 15$.
1 $+$ | 2 | $2x = 81$
2 $\div$ 2 | 3 | $x = 40\frac{1}{2}$ Years, the Wife's Age,

and the Son's Age will be $= 96 - 47 - 42\frac{1}{2} = 8\frac{1}{2}$, the ſame as before.

W. W. D.

PROB-

PROBLEM LI.

Three Merchants from three different Fairs meet together at an Inn, where they reckon up their Gains, and find them the Sum of 780 Crowns. Moreover, if you add the Gain of the firſt and ſecond, and ſubtract the Gain of the third from the Sum, there remains the Gain of the firſt + 82 Crowns; but if you add the Gain of the ſecond and third, and from the Sum ſubtract the Gain of the firſt, there remains the Gain of the third — 43 Crowns. What was the Gain of each?

To anſwer this Query, let x, y, and z repreſent each Merchant's Gain, then ſtating the Queſtion according to the Conditions thereof, it will

$$\begin{array}{rr|l} & 1 & x+y+z=780. \\ \text{Stand thus,} & 2 & x+y-z=x+82. \\ & 3 & y+z-x=z-43. \\ 1 - 3 & 4 & 2x=823-z \\ 1 - 2 & 5 & 2z=698-x \end{array}$$

5 ÷ 2 | 6 | $z = 349 - \frac{x}{2}$, ſubſtitute $349 - \frac{x}{2}$ for z in the fourth Step, and | 7 | $2x = 474 + \frac{x}{2}$

7 $-$ $\frac{x}{2}$ | 8 | $\frac{3x}{2} = 474$

8 ÷ $\frac{3}{2}$ | 9 | $x = 474 \times \frac{2}{3} = 316$ Crowns,

which being known, the third Merchant's Gain will be found at the ſixth Step to be 191 Crowns, and the Sum of theſe two Merchant's Gains, *viz.* 316 + 191 (= 507) ſubtracted from 780 the Sum of all their Gains, gives 273 Crowns the ſecond Merchant's Gain, which was to be found.

NOTE, This Problem may be ſolved by the Help of one unknown Quantity only, which the Learner may try at his Leiſure; but that Method of Computation is too abſtruce for young Beginners, in Propoſitions that are pretty much compounded, as this is. And for this Reaſon I have aſſumed Letters for every Quantity ſought, in ſome Queſtions, purpoſely to render the Calculus more intelligible to the young Analyſt.

PROB-

PROBLEM LII.

Three Perſons, A, B, C, *owe a certain Sum of Money, ſo that* A *and* B *together owe* 210 *Crowns*; B *and* C 290, *and* C *and* A 400. *What did each of them owe?*

By obſerving the Form of this Problem, each Perſon's particular Debt may be readily ſound by one unknown Quantity only. Thus, put $210=a$, $290=b$, $400=c$, $x=$ the Number of Crowns A owes, and (by *Ax.* 1.) $a-x$ will be the Number of Crowns B owes, which taken from b; gives $b-a+x$ the Number of Crowns C owes; now the Sum of x and $b-a+x$ is $b-a+2x$, which by the Queſtion is equal to c, and will ſtand

$$\begin{array}{r|l|l} \text{Thus,} & 1 & b-a+2x=c. \\ 1-b+a & 2 & 2x=c-b+a \\ 2\div 2 & 3 & x=\dfrac{c-b+a}{2}=\dfrac{400-290+210}{2} \end{array}$$

$=160$, the Crowns A owes, which taken from 210 leaves 50, the Crowns B owes, and 50 ſubtracted from 290 there remains 240 the Number of Crowns C owes, which was to be found.

PROBLEM LIII.

To find three Numbers, ſo that the firſt and half of the Remainder, the ſecond and $\frac{1}{3}$ of the Remainder, and the third and $\frac{1}{4}$ of the Remainder, may always make 34.

For the Numbers ſought put x, y, z. And

per Query,	1	$x+\frac{y+z}{2}=a=34.$
	2	$y+\frac{x+z}{3}=a=34.$
	3	$z+\frac{x+y}{4}=a=34.$
1 $\times\overline{2}$	4	$2x+y+z=2a$
2 $\times\overline{3}$	5	$3y+x+z=3a$
3 $\times\overline{4}$	6	$4z+x+y=4a$
4 $-2x-z$	7	$y=2a-2x-z$, ſubſtitute this for y in the
5 & 6 Steps	8	$6a-5x-2z=3a$. Or $5x+2z=3a$
	9	$2a-x+3z=4a$. Or $-x+3z=2a$
9 $+x$	10	$3z=2a+x$
10 $\div\overline{3}$	11	$z=\frac{2a+x}{3}$, which ſubſtitute for z in the

8 Step, | 12 | $\frac{4a+17x}{3}=3a.$

12×3 | 13 | $4a+17x=9a$

$13 - 4a$ | 14 | $17x=5a$

$14 \div 17$ | 15 | $x=\frac{5a}{17}=\frac{5\times 34}{17}=10$, the firſt Number, whence the ſecond Number will be 22, and the third 26. *Q. E. J.*

P R O B L E M LIV.

Let a Square be divided into 9 ſmall Squares: We are to find and diſpoſe the Numbers through the ſeveral ſmall Areas, *ſo that the Sum of every three, taken either laterally or diagonally, may be always* 15.

THIS Queſtion, or an any other of the like Nature is only to be ſolv'd Mechanically, except in thoſe Problems where the Numbers of ſmall Squares are odd, as in this we are about to anſwer; then the middle Numbers will be found Anatycally, but the Poſition of the Reſt Mechanically. Thus let the ſmall Squares be repreſented by the Letters, *a*, *b*, *c*, &c. as the *Fig.* ſpecifies.

a	*b*	*c*
d	*e*	*f*
g	*h*	*m*

Then by the

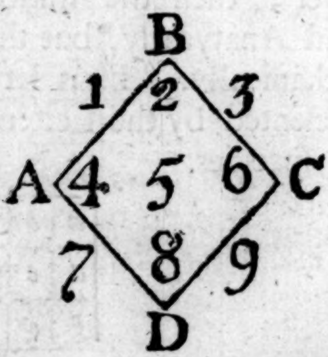
Queſtion,	1	$a+e+m=15$
	2	$b+e+b=15$
	3	$c+e+g=15$
$1+2+3$	4	$a+b+c+3e+m+b+g=45$. Again,
per Query,	5	$a+b+c=15$
	6	$m+b+g=15$
$5+6$	7	$a+b+c+m+b+g=30$.
$4-7$	8	$3e=15$
$8\div 3$	9	$e=5$, the middle Number.

W. W. F.

Now to find the Poſitions of the other Numbers Work mechanically. Thus, ſet down all the Numbers in their progreſſive Order, and they will ſtand

thus, $\left\{\begin{array}{ccc}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{array}\right\}$; and having

drawn the Square ABCD ſo as 1, 3, 7, 9, may fall without;



Then place 1 between 8 and 6, 9 between 4 and 2, 3 between 4 and 8, and 7 between 2 and 6; and there will come

$$\text{out}\left\{\begin{array}{ccc} 2 & 7 & 6 \\ 9 & 5 & 1 \\ 4 & 3 & 8 \end{array}\right\}\text{: The Anſwer}$$

which was required.

If the curious Reader deſire to be more acquainted with the Principles of Magic Squares, let him peruſe *Ronayne*'s *Algebra*.

LV. THEOREM.

Let any Numbers whatſoever be given, if you ſubtract every leſs Number from that which is the next greateſt: I ſay, that the Sum of thoſe Differences is equal to the Difference of the greateſt and leaſt Numbers.

DEMONSTRATION.

Let the Lines *fa*, *hb*, K*c*, *md*, and *oe*, (*See Fig.* 19.) repreſent any Series of Numbers, all drawn from the ſame right Line *ae*, and parallel to each other: and having drawn the pricked Lines *mn*, K*l*, *hi*, and *fg*, all parallel to *ae*, it is evident that *oe* — *md* = *on*, *md* —K*c* = *nl*, K*c* — *bb* = *li*, *bb* — *fa* = *ig*, conſequently the Sum of theſe Differences, *viz.* *on* + *nl* + *li* + *ig* is equal the Difference between the greater Number or Line *oe* and the leſſer Number or Line *fa*, that is, *oe* — *fa* = *on* + *nl* + *li* + *ig*. *Q. E. D.*

PROBLEM LVI.

To find a Number, which being multiplied by 6, *and the Product ſubtracted from the Square of the Number to be found, the Remainder will be* 280.

PUT 280 = *a*, 6 = *b*, and *x* = the Number to be found, whoſe Square is x^2, and multiplied by *b*, the Product is *bx*, which Product ſubtracted from x^2, the Remainder is equal to *a*, by the Queſtion

Therefore,	1	$x^2 - bx = a.$
1 C □	2	$x^2 - bx + \frac{b^2}{4} = a + \frac{b^4}{4}$
2 $\omega\,\overline{2}$	3	$x - \frac{b}{2} = \sqrt{a + \frac{b^2}{4}}.$
3 $+ \frac{b}{2}$	4	$x = \frac{b}{2} + \sqrt{a + \frac{b^2}{4}}. = \frac{6}{2} + \sqrt{280 + \frac{6\times6}{4}}$

= 20, the Number which was to be found.

The

The ſame otherwiſe.

Thus, let $x+3$ be $=$ the Number ſought, whoſe Square is x^2+6x+9, and Product when multiplied by 6 is $6x+18$; whence by the Queſtion

we have | 1 | $x^2+6x+9-6x-18=280$.
That is, 2 $x^2-9=280$.
2 + 9 | 3 | $x^2=289$
3 ω 2 | 4 | $x=17$, and $17+3=20$, the ſame as before. *W. W. F.*

And thus, by aſſuming a compound Quantity for the Number to be found, the reſulting Equation of the above Calculus is reduced to a ſimple Quadratic, which the Learner is deſired to obſerve.

PROBLEM LVII.

To find a Number, which being multiplied 8, *and the Product added to the Square of the Number to be found, the Sum will be* 660.

Put $660=a$, $8=b$, and $x=$ the Number ſought, whoſe Square is x^2, and multiplied by b, the

Product is bx; whence by the Queſtion, we ſhall

have, 1 | $x^2 + bx = a.$
1 C□ 2 | $x^2 + bx + \frac{b^2}{4} = a + \frac{b^2}{4}$
2 ww $\overline{2}$ 3 | $x + \frac{b}{2} = \sqrt{a + \frac{b^2}{4}}.$
3 $- \frac{b}{2}$ 4 | $x = - \frac{b}{2} + \sqrt{a + \frac{b^2}{4}}. = 22$ the

Number which was required.

Or thus,

PUT $x - 4 =$ the Number to be found, the Square of which is $x^2 - 8x + 16$, and 8 times $x - 4$ is $8x - 32$; whence (*per Query*) there will come out this

Equation, 1 | $x^2 - 8x + 16 + 8x - 32 = 660$
That is, 2 | $x^2 - 16 = 660$
2 $+ \overline{16}$ 3 | $x^2 = 676$
3 ww $\overline{2}$ 4 | $x = 26$, and $26 - 4 = 22$ the

ſame as above. *W. W. F.*

PROBLEM LVIII.

To divide 140 *into two Parts, ſo that the Product of thoſe Parts may* = *the Square of* 56, *that is* 3136.

PUT 140 = a, 3136 = b, x = one of the Parts, and (*by Ax.* 1.) $a - x$ will be the other Part; which Parts multiplied together, the Product will be $ax - x^2$, which Product, by the Queſtion, is equal to b.

That is,	1	$ax - x^2 = b.$
1 ±	2	$x^2 - ax = -b$
2 C □	3	$x^2 - ax + \frac{a^2}{4} = \frac{a^2}{4} - b$
3 ɯ $\overset{-}{2}$	4	$x - \frac{a}{2} = \sqrt{\frac{a^2}{4} - b.}$
4 + $\frac{a}{2}$	5	$x = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b.} = \frac{140}{2}$

$\pm\sqrt{\frac{140 \times 140}{4} - 3136.} = 112$, having thus found one of the Parts, ſubtract it from 140 and the Remainder is 28 the other Part:

Q. E. F.

Or

Or thus,

PUT x = the Difference of the two Parts, and (*per Lemma*) the greater Part will be $\frac{a}{2} + \frac{x}{2}$, and the leſſer Part $\frac{a}{2} - \frac{x}{2}$; and the Rectangle of the ſaid Parts $\frac{a^2}{4} - \frac{x^2}{4}$ or $\frac{a^2 - x^2}{4}$, which by the Queſtion, is equal to b.

That is,	1	$\frac{a^2 - x^2}{4} = b$.
1 × 4	2	$a^2 - x^2 = 4b$
2 ±	3	$x^2 = a^2 - 4b$
3 ω 2	4	$x = \sqrt{a^2 - 4b} = \sqrt{140 \times 140 - 4 \times 3136}$

$= 84$. the Difference of the two Parts requir'd, which being known, the greater Part will be $\frac{140}{2} + \frac{84}{2} = 112$, and the leſſer Part $\frac{140}{2} - \frac{84}{2} = 28$, the ſame as before.

PROBLEM LIX.

Let 969 Soldiers be drawn up into an oblong Battle, ſo that the Difference of the greater and leſs Sides is 40. Required the Number of the Soldiers of each Rank in Length and Breadth?

The Import of this Problem is no other than to find two Numbers whoſe Product is 969, and Difference 40. Put 969 $= a$, $40 = b$, $x =$ the Number of Soldiers in Breadth, and the Number of Soldiers in Length (*by Ax.* 2.) will be $x + b$, which multiplied by the Breadth, the Product is $x^2 + bx$; and by the

Queſtion,	1	$x^2 + bx = a$.
1 C □	2	$x^2 + bx + \frac{b^2}{4} = a + \frac{b^2}{4}$
2 ω 2	3	$x + \frac{b}{2} = \sqrt{a + \frac{b^2}{4}}$.
3 $- \frac{b}{2}$	4	$x = - \frac{b}{2} + \sqrt{a + \frac{b^2}{4}} = - \frac{40}{2}$

$+ \sqrt{969 + \frac{40 \times 40}{4}} = 17$; the Number of Soldiers in Breadth, and $40 + 17 = 57$, the Soldiers in Length. *W. W. R.*

Or thus.

PUT $x =$ the Sum of the Soldiers in Length and Breadth, and (*per Lemma*) the Number of Soldiers in Length will be $\frac{x}{2} + \frac{b}{2}$, and in Breadth $\frac{x}{x} - \frac{b}{2}$; which Length and Breadth
2
multiplied by each other, the Product is $\frac{x^2}{4} - \frac{b^2}{4}$, which Product, by the Question, is equal to a.

That is,	1	$\frac{x^2}{4} - \frac{b^2}{4} = a.$
1×4	2	$x^2 - b^2 = 4a$
$2 + b^2$	3	$x^2 = 4a + b^2$
$3\ \omega\ 2$	4	$x = \sqrt{4a + b^2} = \sqrt{4 \times 969 + 40 \times 40} = 74$

the Number of Soldiers in Length and Breadth, whence the Length will be $= \frac{74}{2} + \frac{40}{2} = 57$, and Breadth $= \frac{74}{2} - \frac{40}{2} = 17$. the ſame as before.

PROB-

PROBLEM LX.

Again, let 480 *Soldiers be drawn up into an oblong Battle, ſo that the Sum of the greater and leſs Sides is* 52. *Required the Number of the Soldiers of each Rank in Length and Breadth?*

PUT $480=a$, $52=b$, $x=$one of the Sides of the oblong Battle, and (*by Ax.* 1.) the other Side is $b-x$; which Sides multiplied together the Product is $bx-x^2$, which Product, by the Queſtion, is equal to a, hence the Problem is brought to this

Equation.	1	$bx-x^2=a.$
1 $+$	2	$x^2-bx=-a$
2 C□	3	$x^2-bx+\frac{b^2}{4}=\frac{b^2}{4}-a$
3 ω 2	4	$x-\frac{b}{2}=\sqrt{\frac{b^2}{4}-a}.$
4 $+\frac{b}{2}$	5	$x=\frac{b}{2}\pm\sqrt{\frac{b^2}{4}-a}.=40$, the

Number of Soldiers in Length, and the Number in Breadth will be $52-40=12$, the Ranks which was required.

The ſame otherwiſe.

PUT $x =$ the Difference of the Ranks in Length and Breadth, then (*per Lemma*) the Length will be $\frac{b}{2} + \frac{x}{2}$, and Breadth $\frac{b}{2} - \frac{x}{2}$, whoſe Product when multiplied together is $\frac{b^2}{4} - \frac{x^2}{4}$, which, by the Queſtion, is equal to a.

$$\begin{array}{r|l|l} \text{That is,} & 1 & \frac{b^2}{4} - \frac{x^2}{4} = a. \\ \times\ \overline{4} & 2 & b^2 - x^2 = 4 \\ 1\ \underline{+} & 3 & x^2 = b^2 - 4a \\ 3\ \text{ɯ}\ 2 & 4 & x = \sqrt{\overline{b^2 - 4a}}. = \sqrt{\overline{52\times52 - 4\times480}}. \end{array}$$

$= 28$, the Difference of the Ranks, &c. whence the Length of the Oblong will be $\frac{52}{2} + \frac{28}{2} = 40$ Soldiers, and the Breadth $\frac{52}{2} - \frac{28}{2} = 12$.

W. W. R.

PROBLEM

PROBLEM LXI.

In the Square ABCD *is given the Difference of the Diagonal and the Side, that is* EC = 6. *Required the Side of the Square?*

For AC the Side of the Square (*See Fig.* 20.) put x, and (*by Ax.* 2.) the Diagonal BC = $6 + x$; and, by virtue of the forty-ſeventh Propoſition of the firſt Book of *Euclid*, we ſhall have,

	1	$x^2 + 12x + 36 = 2x^2.$
$1 - x^2 - 12x$	2	$x^2 - 12x = 36$
2 C □	3	$x^2 - 12x + 36 = 72$
3 ω $\overline{2}$	4	$x - 6 = \sqrt{72.} = 6\sqrt{2.}$
4 $+ \overline{6}$	5	$x = 6 - 6\sqrt{2.} = 14.48$, the

Side of the Square. *Q. E. I.*

But by conſidering that all ſquare Planes are ſimilar Figures, this Problem will be eaſily ſolved Numerically. Thus, if the Side of a Square be 1, the Diagonal of that Square will be 1.414+, and conſequently the Difference between the Side and Diagonal will be .414+. Now by Proportion it will be, as .414 : 1 : : 6 : 14. 49 = the Side of the Square. And Univerſally, by dividing the Difference between the Diagonal and Side of any Square, by .414, the Quotient will give the Side of the Square required.

GEOMETRICALLY.

By the forty-ſixth Propoſition of *Euclid*'s firſt Book, make a Square *ab*C*d* (*See Fig.* 21.) of any Magnitude at Pleaſure, and draw the Diagonal *b*C, then with *bd* Radius deſcribe the Arch *de*, and join the Point *d* and *e* with a right Line; this done, on the Diagonal *b*C, ſet off EC = 6, and draw the Line ED parallel to *ed* until it meets with C*d* produced to D. And CD will be the Side of the Square required to be found.

DEMONSTRATION.

Because the Lines *ed* and ED are parallel to each other, the Triangles *e*C*d* and ECD (by the ſecond Propoſition of *Euclid*'s ſixth Book) have their Sides Proportional, whence it will be *e*C : EC : : C*d* : CD, therefore *&c.*

Q. E. D.

PROB.

PROBLEM LXII.

The Rectangle EK *is added to the Square* DF (*being of the ſame Heighth*;) *whoſe Breadth* EL *is given* = 2, *and alſo the Area of the whole compound Rectangle* DK, = 60. *Required the Side of the Square?*

Put 60 = a, 2 = b, and x = ED the Side of the Square (*See Fig.* 22.) then the Area of the ſaid Square will be x^2, and bx the Area of the Rectangle EK, which Areas add together, their Sum is $x^2 + bx$ the Area of the whole compounded Rectangle DK; whence comes out this

Equation,	1	$x^2 + bx = a.$
1 C□	2	$x^2 + bx + \frac{b^2}{4} = a + \frac{b^2}{4}$
2 ww $\overline{2}$	3	$x + \frac{b}{2} = \sqrt{a + \frac{b^2}{4}}.$
3 $- \frac{b}{2}$	4	$x = -\frac{b}{2} + \sqrt{a + \frac{b^2}{4}}. = 6.81$

= ED, the Side of the Square which was to be found.

Or thus.

PUT $x = LD + LK$, and (*per Lemma*) $LD = \frac{x}{2} + \frac{b}{2}$, and $\frac{x}{2} - \frac{b}{2} = LK$; whence, multiplying $\frac{x}{2} + \frac{b}{2}$ by $\frac{x}{2} - \frac{b}{2}$ the Product is $\frac{x^2}{4} - \frac{b^2}{4}$, which by the Queſtion, is equal to the compound Rectangle DK.

$$\begin{array}{r|c|l}
\text{Therefore,} & 1 & \frac{x^2}{4} - \frac{b^2}{4} = a. \\
1 \quad \times 4 & 2 & x^2 - b^2 = 4a. \\
2 \quad + b^2 & 3 & x^2 = 4a + b^2. \\
3 \quad \omega \; 2 & 4 & x = \sqrt{4a + b^2}. = \sqrt{4 \times 60 + 4}.
\end{array}$$

$= 15.62 = LD + LK$, which being known, there will be given $LD = \frac{15.62}{2} + \frac{2}{2} = 8.81.$ and $LK = \frac{15.64}{2} - \frac{2}{2} = 6.81 = ED$, the ſame as before.

PROBLEM LXIII.

A Man buys ſome Ells of Cloth for 70 Crowns; and finds, that if he had 4 Ells more, he had than bought every Ell 2 Crowns cheaper. How many Ells did he buy?

LET $70 = a$, $4 = b$, and $2 = c$. And for the Number of Ells of Cloth put x, and the Price of 1 Ell will be $\frac{a}{x}$ Crowns: But ſuppoſing he had had $x + b$ Ells for a Crown, then 1 Ell at that Rate will coſt $\frac{a}{x+b}$ Crowns, which is 2 or c Crowns leſs than $\frac{a}{x}$. Now, ſince the Difference of theſe two Quotients is 2 or c Crowns, add that Difference to the leſſer Quotient $\frac{a}{x+b}$, to the end that they may become equal, and there will be had this

Equation,	1	$\frac{a}{x+b} + c = \frac{a}{x}$
$1 \times x + b$	2	$a + cx + cb = \frac{ax + ab}{x}$
$2 \quad \times x$	3	$ax + cx^2 + cbx = ax + ab$
$3 \quad - \quad ax$	4	$cx^2 + cbx = ab$

$4 \div c$

4 ÷ c | 5 | $x^2 + bx = \frac{ab}{c}$

5 C $\square$ | 6 | $x^2 + bx + \frac{b^2}{4} = \frac{ab}{c} + \frac{b^2}{4}$

6 ω 2 | 7 | $x + \frac{b}{2} = \sqrt{\frac{ab}{c} + \frac{b^2}{4}}$.

7 $- \frac{b}{2}$ | 8 | $x = -\frac{b}{2} + \sqrt{\frac{ab}{c} + \frac{b^2}{4}}$. $= 10$ the Number of Ells he bought for 70 Crowns, which coſt him $\frac{70}{10} = 7$ Crowns an Ell, for had he had 4 Ells more, then $\frac{10 + 4}{70} = 5$ Crowns would be the Price of an Ell, which is 2 leſs than 7, the Anſwer which was required:

PROBLEM LXIV.

A ſet of boon Companions dining at an Inn the Reckoning in all came to 175 *Shillings: But, before the Bill was paid off, two of them ſlunk away, and then the Club of thoſe that remained came to* 10 *Shillings a Man more. How many were there in Company?*

Put $175 = a$, $10 = b$, and for the Number of boon-Companions put x; then, had not 2 of them ſlunk away, each muſt have paid $\frac{a}{x}$ Shillings towards the Reckoning, but becauſe thoſe that remained cleared the Score, each paid $\frac{a}{x-2}$ Shillings a-piece, which by the Queſtion is 10 or *b* Shillings more than $\frac{a}{x}$.

Now to bring theſe two Quotients into an Equality add 10 or *b* Shillings to the leſſer, and we ſhall have the following

Equation,	1	$\frac{a}{x}+b=\frac{a}{x-2}$
1 $\times\overline{x-2}$	2	$\frac{ax-2a}{x}+bx-2b=a$
2 $\times\ x$	3	$ax-2a+bx^2-2bx=ax$
3 $\pm$	4	$bx^2-2bx=2a$
4 $\div\ b$	5	$x^2-2x=\frac{2a}{b}$
5 C□	6	$x^2-2x+1=\frac{2a}{b}+1$
6 $\omega\ \overline{2}$	7	$x-1=\sqrt{\frac{2a}{b}+1}.$
7 $+\ \overline{1}$	8	$x=1+\sqrt{\frac{2a}{b}+1}.=1+\sqrt{\frac{2\times175}{10}+1}.$

$=7$ the Number in Company, which was required. And had not two of them ſlunk off, each would have paid $\frac{175}{7} = 25$ Shillings, but becauſe 5 paid the Reckoning, they will pay $\frac{175}{5} = 35$ Shillings a piece, which is 10 Shillings more than 25.

PROBLEM XLV.

To divide the Number 21 into two Parts, ſo that if the greater be divided by the leſs, and again the leſs by the greater, and then the firſt Quotient being multiplied by 4, and the latter by 25, the Number produced may be equal.

LET $21 = a$, $4 = b$, $25 = c$, and $x =$ the greater Part, and $y =$ the leſs; then by the

Queſtion. $\left\{ \begin{array}{l|l} 1 & x + y = a. \\ 2 & \frac{bx}{y} = \frac{cy}{x} \end{array} \right\}$

1 $-x$ | 3 | $y = a - x$; which ſubſtituted in the ſecond Step for y, gives

| 4 | $\frac{bx}{a - x} = \frac{ac - cx}{x}$

4 $\times x$ | 5 | $\frac{bx^2}{a - x} = ac - cx$

5 $\times a - x$ | 6 | $bx^2 = a^2c - acx - acx + cx^2$

6 ±	7	$cx^2 - bx^2 - 2acx = - a^2c$
7 ÷ $c - b$	8	$x^2 - \frac{2acx}{c-b} = - \frac{a^2c}{c-b}$
8 in Numbers	9	$x^2 - 50x = - 525$
9 C□	10	$x^2 - 50x + 625 = 100$
10 ω $\overline{2}$	11	$x - 25 = \sqrt{100.} = 10$
11 + $\overline{25}$	12	$x = 25 + 10 = 15$, the greater Part, and $21 - 15 = 6$ the leſſer, the Parts required.

PROBLEM LXVI.

Let the Line AB (*See Fig.* 23.) *be divided in* C, *ſo that* AC *may be* 8, *and* CD 6: *We are to divide the ſame Line* AB *in* D, *ſo that the Rectangle under* AD *and* DC *may be equal to the Rectangle under* AC *and* CB, *or to the Product of* 8 *and* 6, *which is* 48. *Required the Segment* CD?

LET $8 = a$, $6 = b$, $48 = c$, and for the Segment CD put x, and the Segment AD will be $= a + x$. And by the Queſtion we ſhall have | 1 | $x^2 + ax = c.$

1 C$\square$ | 2 | $x^2 + ax + \frac{a^2}{4} = c + \frac{a^2}{4}$

2 $\omega\,\overline{2}$ | 3 | $x + \frac{a}{2} = \sqrt{c + \frac{a^2}{4}}.$

3 $-\frac{a}{2}$ | 4 | $x = -\frac{a}{2} + \sqrt{c + \frac{a^2}{4}}. = -\frac{8}{2} + \sqrt{48 + \frac{8 \times 8}{4}}. = 4$ CD the Segment required.

PROBLEM LXVII.

Let there be a Rectangular Garden ABCD, (*See Fig.* 24.) *the Length of which* AB *is thrice the Breadth* AD: *And reckoning* 18 *Perches from* B *towards* A, *that is* BE, *and drawing* EF *parallel to* AD, *let the Area of the remaining Rectangle* ED *be given* = 120 *ſquare Perches. What was the Length and Breadth of the ſaid Garden?*

PUT $120 = a$, $18 = b$, and $x =$ AE; then AB $= b + x$, and (*by Ax.* 3.) AD $= \frac{a}{x}$. Now ſince by the Queſtion 3AD = AB.

Therefore,

Therefore,		1	$b + x = \frac{3a}{x}$.
1	$\times\, x$	2	$x^2 + bx = 3a$
2	C☐	3	$x^2 + bx + \frac{b^2}{4} = 3a + \frac{b^2}{4}$
3	$\omega\, \overline{2}$	4	$x + \frac{b}{2} = \sqrt{3a + \frac{b}{4}}$.
4	$-\ \frac{b}{2}$	5	$x = -\frac{b}{2} + \sqrt{3a + \frac{b^2}{4}} = 12$.

Hence the Length AB $= 18 + 12 = 30$, and the Breadth AD $= \frac{120}{12} = 10 = \frac{30}{3}$. *Q. E. J.*

PROBLEM LXVIII.

Let 600 *Soldiers be diſpoſed into an oblong Battle; which the Colonel willing to make broader, finds that if he takes away* 10 *Ranks from the Length, he ſhall augment the Breadth with two Ranks. What was the Number of his Soldiers through every Rank in Length and Breadth?*

PUT $600 = a$, $10 = b$ and $2 = c$; and for the Number of Soldiers in Length put x, then (*per Ax.* 3.) the Soldiers in Breadth will be $= \frac{a}{x}$; and by the Queſtion $x - b$ multiplied by $\frac{a}{x} + c$ the Product will be equal to the given Number of Soldiers a; hence the Problem will be brought to the following

Equation,	1	$a + cx - \frac{ab}{x} - bc = a.$
1 $\times$ x	2	$ax + cx^2 - ab - bcx = ax$
2 $\pm$	3	$cx^2 - bcx = ab$
3 $\div$ c	4	$x^2 - bx = \frac{ab}{c}.$
4 C□	5	$x^2 - bx + \frac{b^2}{4} = \frac{ab}{c} + \frac{b^2}{4}$
5 ω $\overline{2}$	6	$x - \frac{b}{2} = \sqrt{\frac{ab}{c} + \frac{b^2}{4}}.$
6 $+ \frac{b}{2}$	7	$x = \frac{b}{2} + \sqrt{\frac{ab}{c} + \frac{b^2}{4}}. = 60$, and ſo

many Soldiers the Colonel placed in the longeſt Rank or Length, which being known, the Number of Soldiers placed in the leſſer Rank or Breadth will be $\frac{600}{60} = 10$; for $60 - 10$ $(= 50)$ multiplied by $10 + 2$ $(= 12)$ the Product is the ſame as 10 times 60, that is, 600.

Q. E. F.

PROB-

PROBLEM LXIX.

A Man buys a Horſe, which he ſells again for 56 Crowns, and gains as many Crowns in 100 *as the Horſe coſt him. How much did he give for the Horſe?*

PUT $100=a$, $56=b$, and $x=$ the Number of Crowns the Horſe coſt him; then (*per Ax.* 1.) he will gain $b-x$ Crowns by ſelling the Horſe again. Now to bring this Queſtion to an Equation, ſay by Proportion, Thus, as $a : x :: x : b-x$: Therefore by multiplying the Means together, and Extremes, there will ariſe this

Equation,	1	$x^2 = ab - ax.$
$1 + ax$	2	$x^2 + ax = ab.$
2 C □	3	$x^2 + ax + \frac{a^2}{4} = ab + \frac{a^2}{4}$
3 $w\,\overline{2}$	4	$x + \frac{a}{2} = \sqrt{ab + \frac{a^2}{4}}.$
$4 - \frac{a}{2}$	5	$x = -\frac{a}{2} + \sqrt{ab + \frac{a^2}{4}}. = 40$

Crowns, and ſo much he gave for the Horſe, which was to be found.

PROBLEM LXX.

A certain Linnen-Draper buys two Sorts of Linnen for 30 Crowns, one finer the other coarſer. An Ell of the fineſt coſt as many Crowns as he had Ells: And alſo 28 Ells of the coarſeſt at ſuch a Price, that 8 Ells coſt as many Crowns as one Ell of the fineſt. How many Ells of the fineſt Linnen did he buy, and what Price did he give for them both?

To ſolve this Problem, put $x=$ the Number of Ells of fine Linnen; and ſince one Ell coſt as many Crowns as he had Ells of that Sort, conſequently x Ells coſt x^2 Crowns; and by the other Condition of the Queſtion, 28 Ells of coarſe Linnen will amount to $\frac{28x}{8}$, or $3.5x$ Crowns, whence $x^2+3.5x$ is the Price of both Sorts of Linnen, which, by the Queſtion, is $=$ to 30 Crowns.

Therefore,	1	$x^2+3.5x=30.$
1 C □	2	$x^2+3.5x+3.0625=33.0625$
2 ω 2	3	$x+1.75=5.75$
3 -1.75	4	$x=4$, the Number of Ells of

fine Linnen, as was to be found, for which he gave 16 Crowns, and the 28 Ells of coarſe Linnen coſt him 14 Crowns. *Q. E. F.*

PROBLEM LXXI.

In a certain Rectangular Garden, (See Fig. 25.) the Length of which AD *is* 22 *Perches, and the Breadth* AB *is* 10, *the Walk* DG *is to be made in a Situation parallel to the Sides of the Figure, ſo that the Area of the ſaid Walk or Gnomon* DG *may be equal to the remaining Rectangle* FC, *or that the Gnomon* DG *may be half of the whole Figure* ABCD *propoſed. Required the Breadth of the ſaid Gnomon* DE, BG?

PUT $22 = a$, $10 = b$, and for the Breadth of the Walk DE put x; then $GB = b - x$, and the Area of the Gnomen GD is $ax + bx - x^2$; which, by the Queſtion, is equal to half the given Parallelogram AC.

$$\begin{array}{ll|l|l}
\text{Therefore,} & & 1 & ax + bx - x^2 = \frac{ab}{2}. \\
1 & \underline{\frac{+}{1}} & 2 & x^2 - ax - bx = -\frac{ab}{2} \\
\underline{2} & \underline{C}\,\square & 3 & x^2 - ax - bx + \frac{a^2}{4} + \frac{ab}{2} + \frac{b^2}{4} = \frac{a^2 + b^2}{4}
\end{array}$$

$\overline{3}$ ɯ $\overline{2}$ | 4 | $x - \frac{a-b}{2} = \sqrt{\frac{a^2+b^2}{4}}. = 9.29$

$4 + \frac{a+b}{2}$ | 5 | $x = 6 \pm 9.29 = 3.29$ Perches,

the Breadth of the Gnomon. *W. W. R.*

PROBLEM LXXII.

Of three proportional Numbers there is the middle Term given = 12, *and the Difference of the Extremes* = 10. *Required the Extremes?*

PUT $12 = a$, $10 = b$, $x =$ the leſſer Extreme, and (*per Ax.* 2.) the greater will be $x + b$; and by the Laws of Geometrical Proportion we ſhall have this

Analogy, | 1 | $x : a :: a : x + b.$

1 *Ergo*, | 2 | $x^2 + bx = a^2$

2 C □ | 3 | $x^2 + bx + \frac{b^2}{4} = a^2 + \frac{b^2}{4}$

3 ɯ $\overline{2}$ | 4 | $x + \frac{b}{2} = \sqrt{a^2 + \frac{b^2}{4}}.$

$4 - \frac{b}{2}$ | 5 | $x = -\frac{b}{2} + \sqrt{a^2 + \frac{b^2}{4}}. = 8$, the

leſſer Extreme, and the greater will be $8 + 10 = 18$, for $8 : 12 :: 12 : 18$, which was to be found.

But becauſe there is given the Difference of the Extremes, let x repreſent their Sum, and (*per Lemma*) the greater Extreme will be $\frac{x}{2}+\frac{b}{2}$, and the leſſer $\frac{x}{2}-\frac{b}{2}$. And from this

Analogy,	1	$\frac{x}{2}+\frac{b}{2}:a::a:\frac{x}{2}-\frac{b}{2}$, we ſhall have this
Equation,	2	$\frac{x^2}{4}-\frac{b^2}{4}=a^2$
2×4	3	$x^2-b^2=4a^2$
$3+b^2$	4	$x^2=4a^2+b^2$
4 ω 2	5	$x=\sqrt{4a^2+b^2}.=26=$ the Sum of

the Extremes, which being found, the greater Extreme will be $\frac{26}{2}+\frac{10}{2}=18$, and the leſſer $\frac{26}{2}-\frac{10}{2}=8$, the very ſame as by the foregoing Method.

PROBLEM

PROBLEM LXXIII.

Of three proportional Numbers there is given the Sum of the firſt and ſecond = 10, *and the Difference of the ſecond and third* = 24. *Required the ſeveral Numbers?*

LET 10 = a, 24 = b, and for the ſecond Number put x; then (*per Ax.* 1.) $a - x =$ the firſt, and the third is $= b + x$. And by the Property of Geometrical Proportion it will

be, as	1	$a - x : x :: x : b + x.$
Ergo,	2	$ab - bx + ax - x^2 = x^2$
2 $\pm$	3	$2x^2 + bx - ax = ab$
3 $\div$ 2	4	$x^2 + \frac{bx - ax}{2} = \frac{ab}{2}$, make $\frac{b - a}{2} = c$.
Then,	5	$x^2 + cx = \frac{ab}{2}$
5 C □	6	$x^2 + cx + \frac{c^2}{4} = \frac{ab}{2} + \frac{c^2}{4}$
6 ω 2	7	$x + \frac{c}{2} = \sqrt{\frac{ab}{2} + \frac{c^2}{4}}.$
7 $- \frac{c}{2}$	8	$x = -\frac{c}{2} + \sqrt{\frac{ab}{2} + \frac{c^2}{4}}. = 8$, the ſe-

cond Number in the Series, whence the firſt will be 10 − 8 = 2, and the third = 24 + 8 = 32; for as 2 : 8 :: 8 : 32. *W. W. R.*

PROBLEM LXXIV.

Of four proportional Numbers there is given the third = 12, *alſo the Sum of the firſt and ſecond* = 8; *beſides the ſecond Number being ſubtracted from its Square, the Remainder is to be the fourth. Required the ſaid Numbers?*

To ſolve this Problem, put 12 = a, 8 = b, and for the ſecond Number put x; then (*per Ax.* 1.) the firſt Number will be $= b - x$, and by the Queſtion, $x^2 - x$ will be the fourth. And from ahe following Analogy, *viz.* $b - x : x :: a : x^2 - x$, there will ariſe, by multiplying the Means and Extremes, this

Equation,	1	$bx^2 - x^3 - bx + x^2 = ax.$
1 +	2	$x^3 - bx^2 - x^2 = -ax - bx$
2 ÷ x	3	$x^2 - bx - x = -a - b$
3 in Numbers,	4	$x^2 - 9x = -20$
4 C☐	5	$x^2 - 9x + 20.25 = .25$
5 ω 2	6	$x - 4.5 = \sqrt{.25.} = .5$
6 + 4.5	7	$x = 4.5 + .5 = 5.$ Having

thus found the ſecond Number, the firſt will be = 8 − 5 = 3, and the fourth = 25 − 5 = 20; for as 3 : 5 :: 12 : 20, the Numbers which was required.

PROBLEM LXXV.

Of four Numbers in continued Proportion there is given the Sum of the Means = 24, *and likewiſe the Sum of the Extremes* = 56. *Required the ſaid Numbers (ſuppoſing that the firſt is the leaſt of all?)*

PUT $24 = 2a$, $56 = b$, and $2x =$ the Difference of the ſecond and third Terms; then (*per Lemma*) $a - x$ will be equal the ſecond Term, and the third will be $a + x$; and from this Analogy, *viz.* as $a - x : a + x :: a + x : \frac{a^2+2ax+x^2}{a-x}$ = the fourth Term, and likewiſe this, *viz.* as $a + x : a - x :: a - x : \frac{a^2-2ax+x^2}{a+x}$ = the firſt, there will ariſe the following Equation,

$$1 \quad \frac{a^2-2ax+x}{a+x}+\frac{a^2+2ax+x^2}{a-x}=b.$$

$$1\times a+x \quad 2 \quad a^2-2ax+x^2+\frac{a^3+3a x+3ax^2+x^3}{a-x}=ab+bx$$

$$2\times a-x \quad 3 \quad \left\{\begin{array}{l}a^3-3a^2x+3ax^2-x^3\\a^3+3a^2x+3ax^2+x^3\end{array}\right\}=a^2b-b$$

3

3 $\pm$ | 4 | $6ax^2 + bx^2 = a^2b - 2a^3$, this Equation in Numbers will ſtand thus,
| 5 | $128x^2 = 4608$
5 $\div$ 128 | 6 | $x^2 = 36$
6 ω 2 | 7 | $x = \sqrt{36.} = 6$, which being found,

the ſecond Term will be $= 12 - 6 = 6$, and the third $= 12 + 6 = 18$; and as 18 : 6 : : 6 : 2 = the firſt Term. Again as 6 : 18 : : 18 : 54 = the fourth, or having found the firſt Term, the fourth will be $= 56 - 2 = 54$, for as 2 : 6 : : 18 : 54. *W. W. R.*

PROBLEM LXXVI.

Two Country-women, A *and* B, *carry* 100 *Eggs together to Market, in the Sale of them, one took as much Money as the other: but* A (*who had the largeſt, and conſequently the beſt Eggs*) *ſays to* B, *had I carried as many Eggs as you, I ſhould have had* 18 *Pence for them;* B *replies, if I had brought as many Eggs as you, I ſhould have had but* 8 *Pence for them. How many Eggs had each?*

To anſwer this Queſtion Univerſally, put $100 = a$, $18 = b$, $8 = c$, and $x =$ the Number of Eggs A carried to Market, and (by *Ax.* 1.) the Number of Eggs B carried will be $a - x$. Now ſince the Eggs A carried to Market were ſold at ſuch a Price, that had they been equal in Number to thoſe B carried, ſhe would have had b Pence for them. Therefore as $a-x : b :: x : \frac{bx}{a-x} =$ the Pence A ſold her Eggs for: and again, had B had the Number of Eggs A carried to Market, ſhe could have made but c Pence of them. Therefore $x : c :: a-x : \frac{ac-cx}{x} =$ the Number of Pence B received for her Eggs. Hence, by the Queſtion, there will be had this,

Equation,	1	$\frac{bx}{a-x}=\frac{ac-cx}{x}$.
$1 \times a-x$	2	$bx=\frac{a^2c-acx-acx+cx^2}{x}$
$2 \quad \times x$	3	$bx^2=a^2c-acx-acx+cx^2$
$3 \quad \pm$	4	$bx^2-cx^2+2acx=a^2c$
$4 \div b-c$	5	$x^2+\frac{2acx}{b-c}=\frac{a^2c}{b-c}$
5 C ☐	6	$x^2+\frac{2acx}{b-c}+\frac{a^2c^2}{b^2-2bc+c^2}$ $=\frac{a^2bc}{b^2-2bc+c^2}$.
$9 \quad \omega \; \overline{2}$	7	$x+\frac{ac}{b-c}=\sqrt{\frac{a^2bc}{b^2-2bc+c^2}}$. $=\frac{a}{b-c}\sqrt{bc}$.
$7-\frac{ac}{b-c}$	8	$x=-\frac{ac}{b-c}+\frac{a}{b-c}\sqrt{bc}.=40,$

the Number of Eggs A carried to Market, and the Number B carried will be $100-40=60$.

Q. E. J.

PROBLEM LXXVII.

Two Country-men, A *and* B, *ſell their Corn at different Prices:* A *ſells* 20 *Buſhels; and* B *received for one Buſhel as many Crowns as he ſold Buſhels:* A *perceives that if he had ſold as many Buſhels as* B *received Crowns, he ſhould then have received* 252 *Crowns; but both together received* 176 *Crowns. How many Buſhels did* B *ſell, and what Price had* A?

PUT $20=a$, $252=b$, and $176=c$, and $x=$ the Number of Buſhels of Corn B ſold, for which, by the Queſtion, he received x^2 Crowns: And ſuppoſing A had ſold x^2 Buſhels of Corn for b Crowns, at the Rate he ſold a Buſhels for. Therefore for a Buſhels he will receive $\frac{ab}{x^2}$ Crowns; which two Sums are equal to c.

That is,		1	$x^2 + \frac{ab}{x^2} = c.$
1	$\times\ x^2$	2	$x^4 + ab = cx^2$
2	$\pm$	3	$x^4 - cx^2 = -ab$
3	C □	4	$x^4 - cx^2 + \frac{c^2}{4} = \frac{c^2}{4} - ab$
4	$\omega\ \overline{2}$	5	$x^2 - \frac{c}{2} = \sqrt{\frac{c^2}{4} - ab}.$
5	$+\frac{c}{2}$	6	$x^2 = \frac{c}{2} \pm \sqrt{\frac{c^2}{4} - ab}.$
6	$\omega\ \overline{2}$	7	$x = \sqrt{\frac{c}{2} \pm \sqrt{\frac{c^2}{4} - ab}} = 6$

the Buſhels of Corn B ſold, for which he received 36 Crowns, and the Price A had for his 20 Buſhels will be found, by what is ſhewn above, to be 140 Crowns, for as 36 : 252 : : 20 : 140.

PROBLEM LXXVIII.

Two Merchants ſell 21 *Ells of Cloth: The firſt ſells* 1 *Ell for as many Crowns, as is* $\frac{1}{5}$ *of the Number of Ells that the ſecond had; and the ſecond ſells* 1 *Ell for as many Crowns, as is* $\frac{1}{3}$ *of the Number of the Ells that they firſt had. The Sale being*

being over, they had taken 48 *Crowns in all. How many Ells did each ſell, and at what Price?*

To ſolve this Queſtion put $21 = a$, $48 = b$, and $x =$ the Number of Ells the firſt Merchant ſold; then (*per Ax.* 1.) $a - x$ will be the Number of Ells the ſecond Merchant ſold, and becauſe the firſt Merchant ſold one Ell for as many Crowns as is $\frac{1}{5}$ of the Number of Ells that the ſecond had. Therefore as $1 : \frac{a-x}{5} :: x : \frac{ax-x^2}{5} =$ the Crowns the firſt Merchant ſold his Cloth for; and again, ſince the ſecond Merchant ſold one Ell for as many Crowns as is $\frac{1}{3}$ of the Number of Ells that the firſt had. Therefore as $1 : \frac{x}{3} :: a - x : \frac{ax-x^2}{3} =$ the Number of Crowns the ſecond Merchant ſold his Cloth for.

Conſequently	1	$\frac{ax-x^2}{5} + \frac{ax-x^2}{3} = b.$
$1 \times \overline{5}$ and $\overline{3}$	2	$8ax - 8x^2 = 15b$
2 $\pm$	3	$8x^2 - 8ax = -15b$
3 $\div$ $\overline{8}$	4	$x^2 - ax = -\frac{15b}{8}$

$$\begin{array}{ll|l|l}
4 & C\,\square & 5 & x^2 - ax + \frac{a^2}{4} = \frac{a^2}{4} - \frac{15b}{8} \\
5 & \omega\ \overline{2} & 6 & x - \frac{a}{2} = \sqrt{\frac{a^2}{4} - \frac{15b}{8}}. \\
6 & + \frac{a}{2} & 7 & x = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - \frac{15b}{8}}. = 15,
\end{array}$$

the Number of Ells the firſt Merchant ſold, and the ſecond ſold 21 — 15 = 6 Ells. Having thus found the Number of Ells of Cloth each ſold, ſay as 1 : $\frac{6}{5}$: : 15 : 18 = the Number of Crowns the firſt Merchant received for his Cloth, and as 1 : $\frac{15}{3}$: : 6 : 30 Crowns, the Price the ſecond Merchant ſold his Cloth at, for 18 + 30 = 48. Which was required.

PROBLEM LXXIX.

Two Merchants have a Parcel of Silk; the firſt 40 Ells, the ſecond 90: The firſt ſells for a Crown $\frac{1}{3}$ of an Ell more than the ſecond: When the Sale was over, they had taken between them 42 Crowns. How many Ells did each of them ſell for a Crown?

For the Number of Ells the firſt Merchant ſold for 1 Crown put $x + \frac{1}{3}$, and the ſecond will ſell x Ells; then as $x+\frac{1}{3}$ Ells : 1 Crown :: 40 Ells : $\frac{40}{x+\frac{1}{3}}$ Crowns, and x Ells : 1 Crown :: 90 Ells : $\frac{90}{x}$ Crowns; which two Sums together are equal to 42 Crowns; whence there will be had the following Equation,

Viz.	1	$\frac{40}{x+\frac{1}{3}}+\frac{90}{x}=42.$
$1\times x+\frac{1}{3}$	2	$40+\frac{90x+30}{x}=42x+14$
2 $\times x$	3	$130x+30=42x^2+14x$
3 $-130x$	4	$42x^2-116x=30$
4 $\div 42$	5	$x^2-2.76x=.7143$
5 C□	6	$x^2-2.76x+1.9044=2.6187$
6 ɯ 2	7	$x-1.38=\sqrt{2.6187}.$
7 $+1.38$	8	$x=2.998$, or rather 3, the

Number of Ells the ſecond Merchant ſold for 1 Crown, whence the firſt Merchant muſt ſell $3\frac{1}{3}$ Ells for the ſame Money.

Q. E. J.

PROBLEM LXXX.

To find a Number, to the quadruple of which if you add 91, *the whole ſhall be to the Square of the Number ſought, as* 3 *to* 4.

To ſolve this Queſtion Univerſally, put 91 $= a$, and for the Ratios 3 and 4 put m and n, likewiſe for the Number ſought put x, then it's Quadruple will be $4x$, and Square $= x^2$; hence by the Queſtion we have this

Analogy,	1	$4x + a : x^2 :: m : n.$
1 *Ergo*,	2	$4nx + an = mx^2.$
2 $-4nx$	3	$mx^2 - 4nx = an$
3 $\div m$	4	$x^2 - \frac{4nx}{m} = \frac{an}{m}$
4 C□	5	$x^2 - \frac{4nx}{m} + \frac{4n^2}{m^2} = \frac{amn + 4n^2}{m^2}$
5 ω $\overline{2}$	6	$x - \frac{2n}{m} = \sqrt{\frac{amn + 4n^2}{m^2}}.$
6 $+\frac{2n}{m}$	7	$x = \frac{2n}{m} + \sqrt{\frac{amn + 4n^2}{m^2}}. = 13.99$

or rather 14, the Number which was to be found.

PROBLEM LXXXI.

To find a Number, from the double of which if you ſubtract 12, *the Square of the Remainder* leſs 1, *will be nine Times the Number ſought.*

PUT 12 = a, 9 = b, and for the Number ſought put x, then the double of x *leſs* a is $2x - a$, whoſe Square is $4x^2 - 4ax + a^2$, from which take 1, and the Remainder, by the Queſtion, is equal to b times x.

That is,	1	$4x^2 - 4ax + a^2 - 1 = bx$.
1 $\pm$	2	$4x^2 - 4ax - bx = -a^2 + 1$
2 ÷ 4	3	$x^2 - ax - \frac{bx}{4} = -\frac{a^2+1}{4}$.
		Subſtitute $2m$ for $-a - \frac{b}{4}$, and n for $-\frac{a^2+1}{4}$.
And	4	$x^2 - 2mx = -n$.
4 C □	5	$x^2 - 2mx + m^2 = m^2 - n$
5 ω 2	6	$x - m = \sqrt{m^2 - n}$.
6 + m	7	$x = m \pm \sqrt{m^2 - n} = 11$, the

Number required. For the Square of 22 — 12 (=10) is 100, and 100 — 1 (=99) is=9×11. *Q. E. F.*

PROB-

PROBLEM LXXXII.

To divide the Number 19 *into two Parts, ſo that the Sum of the Squares of the Parts will be* 193.

PUT $19=a$, $193=b$, and $x=$ one of the Parts; then (*by Ax.* 1.) the other Part will be $a-x$; and the Sum of the Squares of the ſaid Parts is $a^2-2ax+2x^2$, which by the Queſtion is equal to b, hence we

have	1	$a^2-2ax+2x^2=b$.
$1-a^2$	2	$2x^2-2ax=b-a^2$
$2\div\overline{2}$	3	$x^2+ax=\frac{b-a^2}{2}$
3 C □	4	$x^2-ax+\frac{a^2}{4}=\frac{b-a^2}{2}+\frac{a^2}{4}=\frac{2b-a^2}{4}$.
$4\ \omega\ \overline{2}$	5	$x-\frac{a}{2}=\sqrt{\frac{2b-a^2}{4}}$.
$5+\frac{a}{2}$	9	$x=\frac{a}{2}+\sqrt{\frac{2b-a^2}{4}}.=12$, equal

the greater Part, and $19-12=7$ the leſſer, which was to be divided.

Or thus.

Pᴜᴛ $x=$ the Difference of the two Parts, then (by the *Lemma*) $\frac{a}{2}+\frac{x}{2}$ is $=$ the greater Part, and $\frac{a}{2}-\frac{x}{2}$ will be the leſs; and the Sum of Squares of the ſaid Parts will be $\frac{a^2}{2}+\frac{x^2}{2}$, which by the Queſtion is equal to b.

That is, | 1 | $\frac{a^2}{2}+\frac{x^2}{2}=b.$
$1 \times \overline{2}$ | 2 | $a^2+x^2=2b$
$2-\overline{a}$ | 3 | $x^2=2b-a^2$
3 ɯ $\overline{2}$ | 4 | $x=\sqrt{2b-a^2}.=5$ Having thus found the Difference, the Parts are given from what is ſhewn above.

PROBLEM LXXXIII.

To divide 7 *into two Parts, ſo that the Difference of the Squares, which are made from the treble of the leſs Part, and the double of the greater, may be* 17.

Pᴜᴛ $7=a$, $17=b$. and call the greater Part x; then (*per Ax.* 1.) the leſſer will be $=a$

$= a - x$; the treble of the leſſer Part is $3a - 3x$, and double the greater $2x$; which parts being ſquared becomes $9a^2 - 18ax + 9x^2$ and $4x^2$, and $4x^2$ taken from $9a^2 - 18ax + 9x^2$ is equal to $9a^2 - 18ax + 5x^2$, which by the Queſtion, is equal to b. Hence we have this

Equation,	1	$9a^2 - 18ax + 5x^2 = b.$
$1 - 9a^2$	2	$5x^2 - 18ax = b - 9a^2$
$2 \div \overline{5}$	3	$x^2 - \frac{18ax}{5} = \frac{b - 9a^2}{5}$
$3 \; C \; \square$	4	$x^2 - \frac{18ax}{5} + \frac{324a^2}{100} = \frac{20b + 144a^2}{100}$
$4 \; \omega \; \overline{2}$	5	$x - \frac{18a}{10} = \sqrt{\frac{20b + 144a^2}{100}}.$
$5 - \frac{18a}{10}$	6	$x = \frac{18a}{10} \pm \sqrt{\frac{20b + 144a^2}{100}}. = 4,$

which is the greater Part ſought, whence the leſſer will be $= 7 - 4 = 3$, for $3 \times 3 = 9$, and 9 times 9 is 81 : Again $2 \times 4 = 8$, and 8 times 8 is equal 64; take 64 from 81, and the Remainder is 17. *Q. E. J.* and *D.*

PROB-

PROBLEM LXXXIV.

A Man buys a Piece of Linnen, and by ſelling it again, he gains 12 Crowns — $\frac{1}{10}$ *of what he bought it for: And finds by this Means that he had gained as much for 100 Crowns as the Linnen coſt him. What Price was the Linnen bought and ſold at?*

Put $12 = a$, $100 = b$, and $10x =$ the Number of Crowns the Linnen coſt him; then $a - x$ is what he gained by ſelling it again. And from this

Analogy	1	$b : 10x :: 10x : a - x$, we ſhall
have	2	$ab - bx = 100x^2$.
$2 + bx$	3	$100x^2 + bx = ab$, and becauſe b happens to be equal to 100 we ſhall
have	4	$x^2 + x = a$
4 C$\square$	5	$x^2 + x + .25 = a + .25$
5 ω $\underline{2}$	6	$x + .5 = \sqrt{a + .25}$.
$6 - .5$	7	$x = -.5 + \sqrt{a + .25} = 3$, which

multiplied by 10, the Product is 30, the Number of Crowns the Linnen coſt him, and $12 - \frac{30}{10} = 9$ Crowns, is what he gained by ſelling it again, conſequently the Linnen was ſold for 39 Crowns; the Anſwer required.

PROBLEM LXXXV.

A Man buys 18 *Ells of Cloth of different Sorts and Colour, ſuppoſe red and black; what he bought of each coſt* 40 *Crowns: And he pays for every Ell of red Cloth* 1 *Crown more than for the black. How many Ells of each ſort did he buy?*

PUT $40 = a$, $18 = b$, and $x =$ the Number of Ells of black Cloth; then (by *Ax*. 1.) the Number of Ells of red will be $= b - x$. And, ſince he gave a Crowns for each Sort, 1 Ell of black Cloth will coſt $\frac{a}{x}$ Crowns, and $\frac{a}{b-x}$ Crowns will be the Price of 1 Ell of red; but, by the Queſtion, $\frac{a}{b-x}$ exceeds $\frac{a}{x}$ by 1 Crown, therefore add 1 to $\frac{a}{x}$, and there will ariſe this

Equation,

Equation,	1	$\frac{a}{x}+1=\frac{a}{b-x}$.
$1 \quad \times x$	2	$a+x=\frac{ax}{b-x}$
$2 \times b-x$	3	$ab+bx-ax-x^2=ax$
$3 \quad \pm$	4	$x^2+2ax-bx=ab$. Put $2m=2a-b$,
and	5	$x^2+2mx=ab$
$5 \quad$ C□	6	$x^2+2mx+m^2=ab+m^2$
$6 \quad ω \quad \overline{2}$	7	$x+m=\sqrt{ab+m^2}$:
$7-m$	8	$x=-m+\sqrt{ab+m^2}:=10$,

the Number of Ells of black Cloth, and 18 — 10 = 8, the Number of Ells of red.

W. W. R.

PROBLEM LXXXVI.

A Man buys 120 *Pounds of Pepper, and as many of Ginger: and received for a Crown one Pound of Ginger more than of Pepper. So that the whole Price of the Pepper came to* 6 *Crowns more than the Price of the Ginger. How many Pounds of each did he buy for a Crown?*

PUT $120=a$, $6=b$, and $x=$ the Number of Pounds of Pepper he bought for 1 Crown, and then for 1 Crown he had $x+1$ Pounds of Ginger. And by this Analogy, as x Pounds of Pepper : 1 Crown : : a Pounds of Pepper : $\frac{a}{x}$ Crowns $=$ the Price of the Pepper ; and again, as $x+1$ Pounds of Ginger : 1 Crown : : a Pounds of Ginger : $\frac{a}{x+1}$ Crowns, the Price of Ginger. But becauſe the whole Price of the Pepper came to b Crowns more than the Price of Ginger, to the End therefore that they may become equal, add b to $\frac{a}{x+1}$, and the Sum is $\frac{x}{x+1}+b$, or $\frac{a+bx+b}{x+1}$, whence we ſhall

have	1	$\frac{a+bx+b}{x+1}=\frac{a}{b}.$
$1\times\overline{x+1}$	2	$a+bx+b=\frac{ax+a}{x}$
2 $\times x$	3	$ax+bx^2+bx=ax+a$
3 $\pm$	4	$bx^2+bx=a$
4 $\div b$	5	$x^2+x=\frac{a}{b}$
5 C □	6	$x^2+x+.25=\frac{a}{b}+.25$
6 ω $\overline{2}$	7	$x+.5=\sqrt{\frac{a}{b}+.25}.$
7 $-\overline{.5}$	8	$x=-.5+\sqrt{\frac{a}{b}+.25}.=4,$

the Pounds of Pepper bought for 1 Crown, and $4 + 1 = 5$ is the Number of Pounds of Ginger bought for the ſame Money. *Q. E. J.*

PROBLEM LXXXVII.

A Man buys 80 *Pounds of Pepper, and* 36 *Pounds of Saffron, ſo that for* 8 *Crowns he had* 14 *Pounds of Pepper more than he had of Saffron for* 26 *Crowns, and what he had laid out amounted to* 188 *Crowns. How many Pound of Pepper had he for* 8 *Crowns, and how many of Saffron for* 26?

To ſolve this Queſtion, put $x =$ the Number of Pounds of Saffrom bought for 26 Crowns, and the Number of Pounds of Pepper bought for 8 Crowns will be $x + 14$. Hence as x Pounds of Saffron : 26 Crowns : : 36 Pounds of Saffron : $\frac{936}{x}$ Crowns = the Price of 36 Pounds of Saffron. And again, as $x + 14$ Pounds of Pepper : 8 Crowns : : 80 Pounds of Pepper : $\frac{640}{x + 14}$ Crowns = the Price of 80 Pounds of Pepper. But the 80 Pounds of Pepper and 36 Pounds of Saffron, together coſt 188 Crowns by the Queſtion.

Therefore,	1	$\frac{936}{x} + \frac{640}{x + 14} = 188.$
1 $\times x$	2	$936 + \frac{640x}{x + 14} = 188x$
2 $\times x + \overline{14}$	3	$936x + 13104 + 640x = 188x^2 + 2632x.$
3 $\underline{\pm}$	4	$188x^2 + 1056x = 13104$
4 $\div 188$	5	$x^2 + 5.6x = 69.7$
5 C□	6	$x^2 + 5.6x + \underline{7.84} = 77.54$
6 $\omega\ 2$	7	$x + 2.8 = \sqrt{77.54} = 8.8$
7 $-\ 2.8$	8	$x = 6$, the Pounds of Saffron

bought for 26 Crowns, and for 8 Crowns he had 20 Pounds of Pepper, which will be eaſily proved from the Analogies above.

Q. E. J.

PROBLEM LXXXVIII.

A *and* B *between them owe* 174 *Pounds,* A *pays* 8 *Pounds a Day, and* B *pays the firſt Day* 1 *Pound, the ſecond* 2, *the third* 3, *and ſo on. In how many Days will they clear the Debt, and how much did each of them owe?*

PUT 174 = a, 8 = b, and for the Number of Days they required to pay the Debt in put x; then the Number of Pounds A muſt pay of the Debt will be *bx*, and, by the Laws of Arithmetical Progreſſion, multiply the Number of Terms *leſs* 1, *viz.* $x - 1$, by the common Difference of the Series, which is 1, and the Product is $x - 1$ equal the Difference between the two Extremes, to which add 1, and the Sum is x the laſt Term. Now the laſt Term *more* the firſt is $= x + 1$. which Sum multiplied by $\frac{x}{2}$ ($= \frac{1}{2}$ the Number of Terms) the Product will be $\frac{x^2 + x}{2}$, the Sum of all the Series = the Number of Pounds B owes, which added to *bx* the Sum will be $= a$,

That is,	1	$\frac{x^2 + x}{2} + bx = a.$
1 × 2	2	$x^2 + x + 2bx = 2a.$ Put $1 + 2b = 2m.$
And	3	$x^2 + 2mx = 2a$
3 C □	4	$x^2 + 2mx + m^2 = 2a + m^2$
4 ωω 2	5	$x + m = \sqrt{2a + m^2}.$
5 − *m*	6	$x = -m + \sqrt{2a + m^2}. = 12,$

the Days in which A and B paid the Debt, of which, by what is ſhewn above, A owed 96 Pounds, and B 78, the Anſwer required.

Notwithſtanding the foregoing Method of Computation produces an adfected Quadratic

Equation, the Number of Days of Payment may be found by Subtraction only. Thus,

174	—	8 + 1 = 9	=	165	1
165	—	8 + 2 = 10	=	155	2
155	—	8 + 3 = 11	=	144	3
144	—	8 + 4 = 12	=	132	4
132	—	8 + 5 = 13	=	119	5
119	—	8 + 6 = 14	=	105	6
105	—	8 + 7 = 15	=	90	7
90	—	8 + 8 = 16	=	74	8
74	--	8 + 9 = 17	=	57	9
57	—	8 + 10 = 18	=	39	10
39	—	8 + 11 = 19	=	20	11
20	—	8 + 12 = 20	=	0	12

The greateſt Number in the right Hand Column being 12, give the Number of Days of Payment, the ſame as before; which being known, the reſt is obvious.

PROBLEM LXXXIX.

A certain Man intends to travel as many Days as he has Crowns: It happens that every following Day of his Journey he had as many Crowns as he had the Day before, beſides two Crowns over and above: and when he came to his Journey's End he finds he had in all 45 *Crowns. How many Crowns had he at firſt?*

THIS Queſtion imports no more than to find a Series of Numbers in Arithmetical Proportion whoſe Sum is given 45, and common Difference 2; ſuch, that the firſt Term and Number of all the Terms ſhall be equal.

To effect which put $x =$ the Number of Crowns he had in his Pocket when he firſt ſet out, or Number of Days he travel'd, and (by *Corollary* 2. *Chap.* 6. *Part* I, of *Ward's* Introduction) we ſhall have $\overline{x - 1} \times 2 = 2x - 2$ the Difference between the two Extremes, to which add the firſt Term, *viz.* x, and the Sum is $3x - 2$ the laſt Term. Now the Sum of the firſt and laſt Terms, *viz.* $4x - 2$, multiplied by $\frac{x}{2}$ ($= \frac{1}{2}$ the Number of Terms.) the Product will be $2x^2 - x$ the Sum of all the Series, which by the Queſtion is equal to 45 Crowns, hence there comes out this

Equation, 1 $2x^2 - x = 45$.
$1 \div \overline{2}$ 2 $x^2 - \frac{x}{2} = 22.5$
2 C□ 3 $x^2 - \frac{x}{2} + \frac{1}{16} = 22.5 + \frac{1}{16}$
3 ω $\overline{2}$ 4 $x - \frac{1}{4} = \sqrt{\overline{22.5 + \frac{1}{16}}}. = 4\frac{3}{4}$
$4 + \frac{1}{4}$ 5 $x = 5$, the Number of Crowns he had at firſt, for $5 + 7 + 9 + 11 + 13 = 45$.

Q. E. J.

PROBLEM

PROBLEM XC.

A certain Traveller goes 9 Miles a Day, three Days after another follows him, who the firſt Day travels 4 Miles, the ſecond 5, the third 6, and ſo on, gaining a Mile every Day. In what Time will he overtake the former.

Put $x =$ the Number of Days he Travelled that goes 9 Miles a Day, and the Number of Days the other travelled will be $x - 3$; hence he that travels 9 Miles a Day, travelled in all $9x$ Miles, and the other, becauſe he makes his Journey in Arithmetical Proportion, whoſe firſt Term of the Series is 4, and common Difference 1, and likewiſe the Number of Terms $x - 3$, by the Quotation in the foregoing Solution, will travel $\frac{x^2 + x - 12}{2}$ Miles, which muſt conſequently be equal to $9x$, whence ariſe this

Equation,	1	$\frac{x^2 + x - 12}{2} = 9x.$
1 $\times 2$	2	$x^2 + x - 12 = 18x$
2 $\pm$	3	$x^2 - 17x = 12$
3 C$\square$	4	$x^2 - 17x + 72.25 = 84.25$
4 ω 2	5	$x - 8.5 = \sqrt{84.25} = 9.17$
5 $+ 8.5$	6	$x = 17.67$, the Number of Days

he travelled that goes 9 Miles a Days, whence the other Traveller will overtake him at the End of 14.67 Days, the Anſwer required.

PROBLEM XCI.

Two Travellers ſet out at the ſame Time from two Cities, the one from A, *and the other from* B, *which are* 78 *Miles diſtant one from another; one of them goes* 6 *Miles every Day; and the other* 2 *Miles the firſt Day,* 2 *and a half the ſecond, three the third, and ſo on, adding half a Mile to every Day's Journey. In what Time will they meet with one another?*

In order to ſolve this Problem, reduce the Diſtance between the two Cities into half Miles which will be 156; and he that travels 6 Miles a Day will go every Day 12 half Miles, and the other will go 4 half Miles the firſt Day, 5 the ſecond, 6 the third, *&c.* increaſing ½ a Mile every Day 'till he meets with the former. Now ſince they both ſet out from the two Cities at the ſame Time, and proceed towards one another, they will each make his Journey in the ſame Number of Days, which Number call x, then he that travels 12 half Miles a Day, will travel

$12x$ half Miles in x Days, and the other, ſince the Number of half Miles he travels every Day is a Rank of Numbers in Arithmetical Progreſſion, whoſe firſt Term is given $= 4$, common Difference 1, and the Number of Terms $= x$, will make his Journey $\frac{x^2+7x}{2}$ half Miles in the ſame Time, which added to $12x$, the Sum will be equal to the Diſtance between the two Cities; whence the Problem will be reduced to this

Equation,	1	$\frac{x^2+7x}{2}+12x=156$.
1 × $\overline{2}$	2	$x^2+31x=312$
2 C□	3	$x^2+31x+240.25=552.25$
3 ɯ 2	4	$x+15.5=\sqrt{552.25}=23.5$
4 − 15.5	5	$x=8$, the Number of Days they

muſt travel before they meet.

W. W. R.

Or thus.

Add together ſucceſſively the Number of Miles they both travel every Day, until the Sum amounts to 78 Miles the Diſtance between the two Cities A and B, and the Number of ſuch Additions will give the Anſwer required. See the Operation.

Days.			Miles.
1	$6 + 2$	=	8.0
2	$6 + 2.5 + 8$	=	16.5
3	$6 + 3 + 16.5$	=	25.5
4	$6 + 3.5 + 25.5$	=	35.0
5	$6 + 4 + 35$	=	45.0
6	$6 + 4.5 + 45$	=	55.5
7	$6 + 5 + 55.5$	=	66.5
8	$6 + 5.5 + 66.5$	=	78.0

SEEING in the left Hand Column the Number of Additions are 8, they therefore met each other after eight Days Travel. The ſame as before.

PROBLEM XCII.

Again, Two Travellers ſet out at the ſame Time from two Cities, the one from A, *and the other from* B, *which are* 120 *Miles diſtant from one another; the firſt goes* 5 *Miles a Day, and the other* 3 *Miles leſs than the Number of Days in which they meet. When will they meet?*

PUT $x =$ the Number of Days in which they meet, and the Number of Miles the ſecond Traveller goes every Day will be $x - 3$. And they will meet one another after the firſt has gone $5x$ Miles, and the other $x^2 - 3x$ Miles.

Whence,

Whence,	1	$x^2-3x+5x=120.$
Or,	2	$x^2+2x=120$
2 C □	3	$x^2+2x+1=121$
$3\ \omega\ \underline{2}$	4	$x+1=\sqrt{121}.=11$
$4-1$	5	$x=10$ Days, the Time required.

PROBLEM XCIII.

A Poſt ſets out from A *towards* B, *who travels* 8 *Miles a Day: After he had gone* 27 *Miles, another ſets out from* B *to meet him, who goes every Day* $\frac{1}{20}$ *of the whole Journey, or Diſtance of the two Places* A *and* B, *and meets the firſt Poſt after ſo many Days as is* $\frac{1}{20}$ *of the ſaid Diſtance. Required the Diſtance of* A *and* B?

For the Diſtance between the two Places A and B put $20x$, and the ſecond Poſt will go every Day x Miles; and becauſe he met the firſt after he had travelled as many Days as was an x Part of the whole Diſtance, he will make his Journey x^2 Miles in x Days; and the firſt Poſt will make his Journey to the Place where they meet $8x+27$ Miles.

Whence,

Whence,	1	$x^2 + 8x + 27 = 20x$.
1 $\pm$	2	$x^2 - 12x = -27$
2 C $\overline{\square}$	3	$x^2 - 12x + 36 = 9$
3 ω $\overline{2}$	4	$x - 6 = 3$
4 $+ \overline{6}$	5	$x = 9$. conſequently $20x = 180$ Miles,

the Diſtance between A and B. Which was required.

PROBLEM XCIV.

Two Merchants A *and* B *go Partners,* B *brings* 420 *Crowns, and* A *receives out of the Gains* 52 *Crowns, and the Sum of both their Shares is* 854 *Crowns. How much did* A *bring, and how much did* B *receive out of the Gains?*

To anſwer this Problem put $420 = a$, $52 = b$, $854 = c$, and for the Stock A put in put x; then as x (=A's Stock) $:$ b (what he received out of the Gains) $::$ a (= the Number of Crowns B brought in): $\frac{ab}{x}$ = B's Gain; and by adding both the Merchants Shares together, the Sum, *viz.* $x + \frac{ab}{x} + a + b$, will be equal to c, hence we have the following

Equation

Equation,	1	$x+\frac{ab}{x}+a+b=c.$
1 $\times x$	2	$x^2+ab+ax+bx=cx$
2 $\pm$	3	$x^2+ax+bx-cx=-ab$
3 in Numb.	4	$x^2-382x=-21840$
4 C □	5	$x^2-382x+\underline{36481}=14641$
5 ω 2	6	$x-191=\sqrt{14641.}=121$
6 $+$ 191	7	$x=312$ the Numb. of Crowns

A put into the Stock. And as 312 : 52 : : 420 : 70 = the Number of the Crowns B received out of the Gain. *W. W. R.*

PROBLEM XCV.

A Son asks his Father how old he was? his Father replied thus; If you take 4 *from my Age, the Remainder will be thrice the Number of your Years: But, if you take* 1 *from your Age, half the Remainder will be the Square Root of my Age. Required the Age of the Father and Son?*

Put x = the Father's Age, and the Son's Age will be $=\frac{x-4}{3}$ Years, now divide $\frac{x-4}{3}-1$ by 2, the Quotient is $\frac{x-4}{6}-\frac{1}{2}$, which, by the Queſtion, is equal to the ſquare Root of x.

Whence, | 1 | $\frac{x-4}{6}-\frac{1}{2}=\sqrt{x}.$

1 ⊙ $\overline{2}$ | 2 | $\frac{x^2-8x+16}{36}-\frac{x+4}{6}+\frac{1}{4}=x$

2 $\overline{\times 36}$ | 3 | $x^2-8x+16-6x+24+9=36x$

3 ± | 4 | $x^2-50x=-49$

4 C □ | 5 | $x^2-50x+\underline{625}=576$

5 ww $\overline{2}$ | 6 | $x-25=\sqrt{576}.=24$

6 + 25 | 7 | $x=49$ Years the Father's Age, and the Son's Age, by what is ſhewn above, will be found to be 15 Years. *W. W. R.*

PROBLEM XCVI.

To find two Numbers, the Sum of whoſe Squares may be 317, *and the Product, if they may be multiplied by one another*, 154.

PUT $317=a$, $154=b$, and $x=$ the greater Number; then (*per Ax.* 3.) $\frac{b}{x}$ will be the leſſer, and by the

Queſtion, | 1 | $x^2+\frac{b^2}{x^2}=a.$

1 × x^2 | 2 | $x^4+b^2=ax^2$

2 ± | 3 | $x^4-ax^2=-b^2$

$$\begin{array}{l l | l | l}
3 & C\,\square & 4 & x^4 - ax^2 + \frac{a^2}{4} = \frac{a^2}{4} - b^2 \\
4 & \text{ɯ}\,\overline{2} & 5 & x^2 - \frac{a}{2} = \sqrt{\frac{a^2}{4} - b^2}. \\
5 & +\frac{a}{2} & 6 & x^2 = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b^2}. \\
6 & \text{ɯ}\,\overline{2} & 7 & x = \sqrt{\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b^2}} = 14,\ \text{the}
\end{array}$$

greater Number, and the leſſer will be $\frac{154}{14} = 11$; for the Square of 14 is 196, of a 11 121, and $196 + 121 = 317$. *Q. E. I.* and *D.*

The ſame otherwiſe.

THUS, put $\left\{\begin{array}{l} x = \text{Sum} \\ y = \text{Diff.} \end{array}\right\}$ of the two Numbers, and (*per Lemma*) the greater Number will be $\frac{x}{2} + \frac{y}{2}$, and the leſſer $= \frac{x}{2} - \frac{y}{2}$; which being ſtated according to the Condition of the Queſtion, will ſtand

$$\text{Thus}\left\{\begin{array}{l|l}
1 & \frac{x^2}{4} - \frac{y^2}{4} = b. \\
2 & \frac{x^2}{4} + \frac{xy}{2} + \frac{y^2}{4} + \frac{x^2}{4} - \frac{xy}{2} + \frac{y^2}{4} = a
\end{array}\right\}$$

2 Con-

2 Contracted	3	$\frac{x^2}{2}+\frac{y^2}{2}=a$
1 $\times\overline{2}$	4	$\frac{x^2}{2}-\frac{y^2}{2}=2b$
3+4	5	$x^2=a+2b$
5 ω $\overline{2}$	6	$x=\sqrt{a+2b}.=25.$ which being

known, the Numbers required will be found from what is ſhewn above.

But if for the leſſer Number be put x, and xy for the greater. Then by the

Queſtion,	1	$x^2y=b.$
	2	$x^2+x^2y^2=a.$
1 $\div y$	3	$x^2=\frac{b}{y}$ which ſub. in the 2d Step
for x^2, gives	4	$\frac{b}{y}+\frac{by^2}{y}=a$
4 $\times y$	5	$b+by^2=ay$
5 $\pm$&$\div$	6	$y^2-\frac{ay}{b}=-1$
6 C □	7	$y^2-\frac{ay}{b}+\frac{a^2}{4b^2}=\frac{a^2}{4b^2}-1$

7	$\omega\ \bar{2}$	8	$y - \frac{a}{2b} = \sqrt{\frac{a^2}{4b^2} - 1.} = .24$
8	$+ \frac{a}{2b}$	9	$y = \frac{a}{2b} + \sqrt{\frac{a^2}{4b^2} - 1.} = 1.27$ the

Value of y being thus found, x will be known from this Equation $x^2 = \frac{b}{y}$, (See Step the third) for the Root being extracted, gives $x = \sqrt{\frac{b}{y}}. = 11.01$ or rather 11, the leſſer Number, and the greater Number will be $xy = 11 \times 1.27 = 13.97$ or rather 14. the ſame as by the foregoing Operations.

This Method of Computation I never met with in any Author; nor will I preſume to ſay it is my own Invention; for it was firſt communicated to me by my moſt ingenious and worthy Friend Mr *Samuel Waters*. But how he came by it, or whether it was his own Contrivance, I am at a loſs to determine.

PROBLEM

PROBLEM XCVII.

To find two Numbers, the Product of which may be 180, *and the Difference of the Squares* 63.

PUT $108 = a$, $63 = b$, and call the greater Number x; then (*per Ax.* 3.) the leſſer Number will be $\frac{a}{x}$, and ſince the Difference of the Squares of the ſaid Numbers is to be equal to b we ſhall have this

Equation,	1	$x^2 - \frac{a^2}{x^2} = b.$
1 $\times x^2$	2	$x^4 - a^2 = bx^2$
2 $\pm$	3	$x^4 - bx^2 = a^2$
3 C$\square$	4	$x^4 - bx^2 + \frac{b^2}{4} = a^2 + \frac{b^2}{4}$
4 $\omega\ \overline{2}$	5	$x^2 - \frac{b}{2} = \sqrt{a^2 + \frac{b^2}{4}}.$
5 $+\frac{b}{2}$	6	$x^2 = \frac{b}{2} + \sqrt{a^2 + \frac{b^2}{4}}.$
6 $\omega\ \overline{2}$	7	$x = \sqrt{\frac{b}{2} + \sqrt{a^2 + \frac{b^2}{4}}}\ .. = 12,$ the

greater Number, and $\frac{108}{12}$ gives 9 the leſſer.

Q. E. J.

The

The ſame otherwiſe.

THUS, put $\left\{\begin{matrix} x = \text{leſſer} \\ xy = \text{greater} \end{matrix}\right\}$ Number, and Difference of their Squares will be $x^2y^2-x^2$, and Product x^2y.

Whence,	1	$x^2y=a.$ } by the Queſtion.
	2	$x^2y^2-x^2=b.$ } by the Queſtion.
$2 \div \overline{y^2-1}$	3	$x^2=\frac{b}{y^2-1}$. which ſubſtitute for x^2 in the firſt
Step, and	4	$\frac{by}{y^2-1}=a$
$4 \times \overline{y^2-1}$	5	$by=ay^2-a$
$5 \; \underline{\pm} \; \& \div$	6	$y^2-\frac{by}{a}=1$
6 C□	7	$y^2-\frac{by}{a}+\frac{b^2}{4a^2}=\frac{b^2}{4a^2}+1$
7 ɯ $\overline{2}$	8	$y-\frac{b}{2a}=\sqrt{\frac{b^2}{4a^2}+1.}=1.03$
$8 + \frac{b}{2a}$	9	$y=\frac{b}{2a}+\sqrt{\frac{b^2}{4a^2}+1.}=1.31+.$

Having thus found y, x may be known from any of the three firſt Steps; which the Learner may find at his Leiſure.

PROB

PROBLEM XCVIII.

Two Farmers ſell two ſorts of Corn: A ſells 6 Buſhels; B receives in all for his 20 Crowns: Now, ſays B to A, if we add the Number of my Buſhels to the Number of your Crowns, the Sum will be 28. Says A to B, and if I add the Square of my Crowns to the Square of your Buſhels, the Sum will be 424. How many Buſhels did B ſell, and how many Crowns did A receive?

LET $6 = a$, $20 = b$, $28 = c$, $424 = d$, and for the Number of Buſhels B ſold put x; then $c - x$ will be the Number of A's Crowns, whoſe Square is $c^2 - 2cx + x^2$, and the Square of B's Buſhels will be x^2; which Squares added together, their Sum is $c^2 - 2cx + 2x^2$ which, by the Queſtion, is equal d; whence ariſes this

Equa-

Equation,	1	$c^2-2cx+2x^2=d$
1 $-c$	2	$2x^2-2cx=d-c^2$
2 $\div\overline{2}$	3	$x^2-cx=\dfrac{d-c^2}{2}=-180$. For c^2 is greater than d.
3 C□	4	$x^2-cx+\dfrac{c^2}{4}=\dfrac{d-c^2}{2}+\dfrac{c^2}{4}$
		$=\dfrac{2d-c^2}{4}$
4 $\omega\,\overline{2}$	5	$x-\dfrac{c}{2}=\sqrt{\dfrac{d-c^2}{2}+\dfrac{c^2}{4}}.$
		$=\sqrt{\dfrac{2d-c^2}{4}}.$
5 $+\dfrac{c}{2}$	6	$x^2=\dfrac{c}{2}\pm\sqrt{\dfrac{d-c^2}{2}+\dfrac{c^2}{4}}.=\dfrac{c}{2}$
		$\pm\sqrt{\dfrac{2d-c^2}{4}}.$

= 18 the Number of Buſhels B ſold, which taken from 28, will leave 10 the Number of Crowns A received. *W. W. R.*

PROB.

PROBLEM CXIX.

To find two Numbers, the firſt of which $+ 2$ *multiplied into the ſecond* $- 3$, *may produce* 110: *and on the contrary, the firſt* $- 3$, *multiplied by the ſecond* $+ 2$, *may produce* 80.

For the firſt Number put x, and y the ſecond, then $x+2$ multiplied into $y-3$, and $x-3$ multiplied into $y+2$ produces the following

Equations,	1	$xy-3x+2y-6=110.$
	2	$xy+2x-3y-6=80.$
1 $+3x+\overline{6}$	3	$xy+2y=116+3x$
2 $-2x+\overline{6}$	4	$xy-3y=86-2x$
3 $\div\ \overline{x+2}$	5	$y=\dfrac{116+3x}{x+2}$
4 $\div\ \overline{x-3}$	6	$y=\dfrac{86-2x}{x-3}$
5 = 6	7	$\dfrac{116+3x}{x+2}=\dfrac{86-2x}{x-3}$
7 $\times\ \overline{x+2}$	8	$116+3x=\dfrac{86x-2x^2+172-4x}{x-3}$
8 $\times\ \overline{x-3}$	9	$116x+3x^2-348-9x=86x-2x^2+172-4x$

9	$\pm$	10	$5x^2+25x=520$
10	$\div 5$	11	$x^2+5x=104$
11	C□	12	$x^2+5x+6.25=110.25$
12	ω 2	13	$x+2.5=\sqrt{110.25}=10.5$
13	-2.5	14	$x=10.5-2.5=8$, the first

Number, and, by the 5th or 6th Step, the Second will be fouud to be 14 : for $8+2\times 14-3=110$. and $8-3\times 14+2=80$.

Q. E. I.

Or thus.

LET x repreſent the firſt Number as above, and $\frac{110}{x+2}$ is equal the ſecond *leſs* 3, therefore $\frac{110}{x+2}+3$ will be the ſecond Number, to which add 2, and the Sum is $\frac{110}{x+2}+5$, which Sum multiplied by $x-3$ the Product will be $\frac{110x-330}{x+2}+5-15$, and the ſaid Product, by the Queſtion, is equal to 80, hence is obtained the following

Equation,		1	$\frac{110x-330}{x+2}+5x-15=80.$
1	$\times x+\overline{2}$	2	$110x-330+5x^2-5x-30=80x+160$
2	$\underline{\pm}$	3	$5x^2+25x=520$
3	$\div\ \overline{5}$	4	$x^2+5x=104$
4	C $\underline{\square}$	5	$x^2+5x+6.25=110.25$
5	ɯ $\underline{2}$	6	$x+2.5=\sqrt{110.25.}=10.25$
6	-2.5	7	$x=10.5-2.5=8$ the firſt Number,

and $\frac{110}{8+2}+3=14$ the ſecond, the ſame as before.

Having now finiſhed theſe XCIX Queſtions and Anſwers, further to excite the Genius of the young Analyſt, I ſhall conclude theſe few Pages with the following Problem, but leave the Solution to employ his leiſure Hours.

PROBLEM C.

Suppoſe a Cask holds 81 *Gallons of Wine when full, out of which a certain Quantity is exhauſted, and then the Cask is filled up again with Water; the ſame Quantity being again drawn out as at firſt, and the Cask again filled up with Water, and ſo on four Times (always filling the Cask with Water after every Evacuation) there is at laſt found* 16 *Gallons of Wine left in the Cask beſides Water. The Queſtion is, what Quantity of Wine was drawn out each Time?*

29 OC 62

FINIS.

Printed by C. Jephson, in *Wine-Office-*
Court, *Fleet-Street*.